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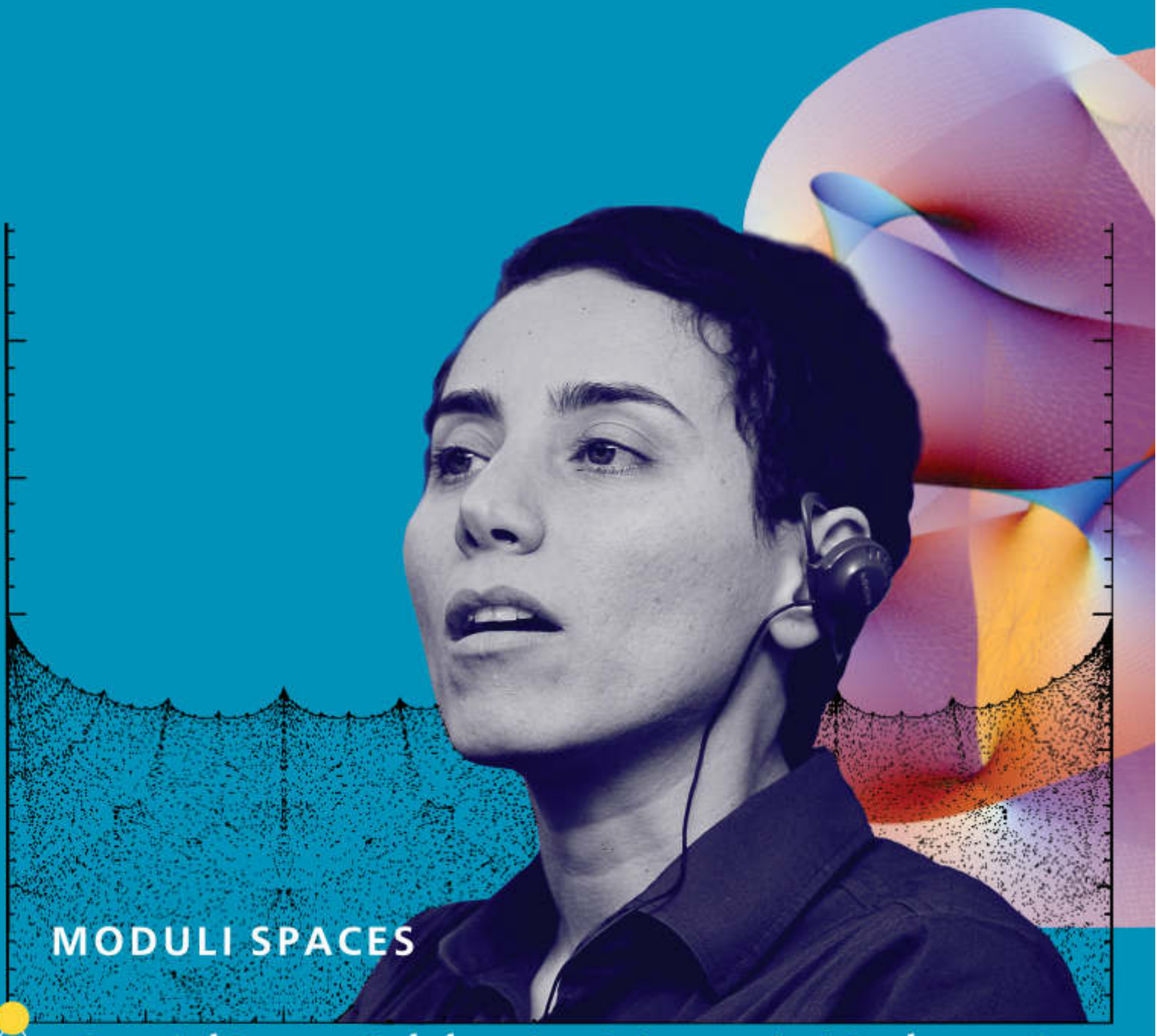

Special Issue: Celebrating Women in Mathematics

Celebrating the Life & Works of Maryam Mirzakhani

MSI Newsletter

Vol II - Number 1

Special Issue: Celebrating Women in Mathematics

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And we also have an obituary article on S. S. Shrikande a combinatorial mathematician.

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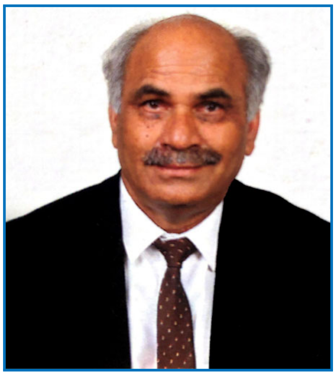
In this issue we have come up with a new series, "Living Legends" where we describe some internationally acclaimed mathematicians. Prof. Peter Sarnak of Princeton University (IAS) and Ingrid Daubechies of Duke University are the two mathematicians featured here.

Ingrid Daubechies

The MSI Samplers

From the Desk of Director, MSI



T. Venkatesh, Director, MSIB

This issue of our Newsletter is on the occasion of Women in Mathematics WIM celebrations and hence we have a feature article on the first ever women mathematician to receive the Fields medal (considered to be one of the highest awards in Mathematics). May 12th initiative brings together virtual or local events celebrating women in mathematics. The initiative was first proposed at the world meeting for Women in Mathematics (WM)^2 in 2018.

The article authored by J .V. Ramana Raju chronicles the life and works of Maryam Mirzakhani whose life was cut short by the unfortunate event of her death due to cancer at the age of 40. We also have obituary notes on two of the senior most mathematicians that India has produced. One is on C. S. Seshadri the great algebraic-geometer and Institution builder and another one is on the Combinatorial Mathematician S. S. Shrikhande. We also note that we lost quite a number of acclaimed senior mathematicians in recent times including V. S. Varadarajan.
We have added newly 'the classics' section where we showcase a seminal work or a very ground breaking result. There is a quick review of what the Institute was doing in the last few months, in the section entitled MSI Roundup. Inspite of the pandemic throwing up challenges, the Institute has been consistent in its outreach activities and just a few weeks ago we initiated the PU Math Lectures as an outreach effort. To encourage young people do research the Institute has initiated the Research Awards for MSI interns - RAMI. Each year two candidates will be chosen and they shall complete a mini-project and present it to the Institute. This year Pooja B of Vidhyagiri College of Arts and Science, Karaikudi. and Vidyasagar Mysoremath of RCU, Belagavi have been chosen for this award. I sincerely hope this news shall inspire the young readers.

Prof. K. B. Athreya is our good friend who regularly visits Belagavi, and hence we have come up with his interview in our face2face section.

Two interesting books have been reviewed by out editorial board members, one is the classical book "What is Mathematics" written by Richard Courant and Herbert Robbins, and another is a relatively recent book "An Invitation to Algebraic Geometry" by Smith Karen, Lauri Kahanpaa, Kekalainen Pekka, William Traves.

I hope the readers would enjoy reading the contents of this issue, and as always we welcome suggestions. We expect more participation in our activities, from math-enthusiasts across the globe. I wish you all the best, let us stay connected.

- T.Venkatesh, Director, MSIB

Feature Article

The Life and Works of Maryam Mirzakhani

In a span of 40 years of her shortened life Maryam Mirzakhani the first ever woman mathematician who received the Fields medal led a life by example. She was always compassionate toward the downtrodden and also very considerate and accomodative to young researchers especially for the women commmunity. Here we feature some glimpses of her mathematical life in this special issue.

We pay tributes to one of the finest mathematicians of modern times Maryam Mirzakhani who died at an early age after she became the first ever woman field medalist. Her outstanding mathematical ability became evident when she won gold medals in the International Mathematical Olympiads in HongKong(1994) and again in Canada(1995), becoming the only Iranian ever to achieve a perfect score.

Born in May 1977 in Tehran(Iran), right from her early student days she won several prizes in the sciences. She won a place at Farzanegan secondary school, for exceptionally talented students. Here they say, she had inspiring teachers and friends. Supported by her head teacher, Maryam entered mathematical competitions previously reserved only for boys and represented Iran at the IMO and as recorded it has become a history. Her performance was so impressive that she received permission to skip the national qualifying exam for university admission. She gained her bachelor's degree at Sharif University in Tehran, and in 1999 she moved to Harvard, where she studied under the direction of Curt McMullen also a Fields medallist. In 2018, the American Mathematical Society governance developed and approved two ways by which to commemorate Maryam Mirzakhani and her mathematical legacy. The Maryam Mirzakhani Lecture will be an annual AMS Invited Address at the Joint Mathematics Meetings (JMM). The lecture will allow leading scholars to present their research, while commemorating the exceptional accomplishments of Maryam Mirzakhani. 'The Maryam Mirzakhani Fund for The Next Generation' is an endowment that exclusively supports programs for early career mathematicians; i.e., doctoral and postdoctoral scholars. It is part of a special initiative and aims to assist rising scholars each year at modest but impactful levels. In what follows we shall try to narrate several instances of her academic and public life in an effort to inspire the young.

Marayam Mirzakhani *Photo credits AMS.org*

First we enlist in verbatim a few citations which can give us an idea of the breadth of her work. Maryam won the Clay Research Award in recognition of her "many and significant contributions to geometry and ergodic theory, in particular to the proof of an analogue of Ratner's theorem on unipotent flows for moduli of flat surfaces". Earlier in 2004 the Clay Math Institute awarded her the Clay Research fellowship for a period of four years. The International Mathematical Union in the Fields medal award citation said "she is awarded the Fields Medal for her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces, further the citation said Maryam Mirzakhani has made stunning advances in the theory of Riemann surfaces and their moduli spaces, and led the way to new frontiers in this area. Her insights have

Feature Article continued...

integrated methods from diverse fields, such as algebraic geometry, topology and probability theory. At the Blumenthal award 2009 ceremony the citation said, Mirzakhani is honored" for her exceptionally creative, highly original thesis. This work combines tools as diverse as hyperbolic geometry, classical methods of automorphic forms, and symplectic reduction to obtain results on three different important questions. These results include a recursive formula for Weil-Petersson volumes of moduli spaces of Riemann surfaces, a determination of the asymptotics of the number of simple closed geodesics on a hyperbolic surface in terms of establishing the KdV recursion for the intersection numbers on moduli space." While at Princeton, Maryam met Jan Vondr a k, a Czech national who is a doctorate in computer science and also a PhD in applied mathematics from the Massachusetts Institute of Technology. Jan was then a post-doctoral teaching fellow at Princeton. They married in 2005 and had a daughter Anahita in 2011. Her characteristic personality involving resilience, hard work, and humility is seen as an example for youngsters dreaming to make it big in the world of science. She was a gentle humble person and a great teacher as well as a supervisor. Maryam once said, in a rare interview: "I can see that, without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers". She was known to her colleagues as "a virtuoso in the dynamics and geometry of complex surfaces" but to her 6-year-old daughter (then), she was "something of an artist" who used to draw things on paper for hours in an enthusiastic attempt to discover formulae that can describe the complexities of curved surfaces. Elon Lindenstrauss a mathematician working at the Einstein Institute of Mathematics who has fond memories of working with Maryam both at the IAS Princeton and at Berkeley for a semester long special program says "she often exceeded the expectations(of her peers), because she was not trying to reap low-hanging fruits". If anyone has reached this far he/she would have definitely got glimpses of the kind of exemplary academic work that Maryam has put forth. We have deliberately tried not to over simplify the description of her work since there are many such descriptions in online magazines like the Scientific American and the article by Gareth Cook in NYT.

Marayam & Family *Photo credits AMS.org*

Her work concerns geometrical objects and their classifying spaces. It is about Riemann surfaces and their moduli spaces. Her PhD thesis question was to find certain numbers associated to a family of geodesic curves that represent Riemann surfaces. These numbers in fact represent volumes, to be more precise the Weil-Peterson volume of moduli spaces of certain special type of Riemann surfaces and also in another direction an asymptotic count of the number of simple closed geodesics of bounded length on certain esoteric geometric objects called as hyperbolic surfaces.

Feature Article continued...

Soon she announced that part of her earlier work can be used to give a new proof of the famous Witten-conjectures which she did. The magnanimity of her works can be gauged by the fact that her first paper regarding the volumes was published in Inventiones Mathematicae and her other results of the thesis were published in in the Annals of Mathematics Journal and Journal of the American Mathematical Society all of them being very prestigious and respected journals for the Math-Community. Her later work refers to some orbital dynamics results in billiard type spaces where a point mass moves in a closed space and traces several geodesics through reflections. In some cases, it is helpful to embed the space of certain billiard tables in a larger space in which every point is a surface that is locally either flat or cone-shaped. With Alex Eskin, a mathematician at the University of Chicago at Illinois, Mirzakhani used this method to prove, for such spaces, a version of a theorem about a group of symmetric geometric objects known as Lie groups.

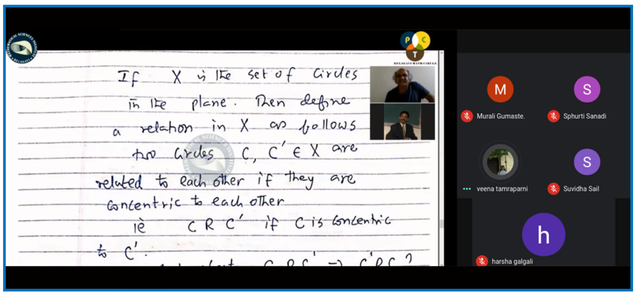
The theorem was proposed by Marina Ratner, another towering woman mathematician in the field who also died in the same year, July 2017 at the age 78. The proof a monumental work written up in a 200-page paper (A. Eskin and M. Mirzakhani Preprint at `https://arxiv.org/abs/1302.3320`; 2013) - tied together disparate fields including geometry, topology and dynamical systems, and spanned a field of its own. It has been dubbed the 'magic wand' theorem because it enabled many previously intractable mathematical problems to be solved. For instance they worked in certain geometric spaces called translation surfaces and gave an understanding of the dynamics of Teichmuller flows. Talking about the Teichmuller spaces Secrets of Surface: The Mathematical Vision of Maryam Mirzakhani is a short documentary prepared by MSRI which is Filmed in Canada, Iran, and the United States, Apart from Mathematics she was also an inspiration to young women everywhere according to the words of her husband Jan. And so her life is an immense source of encouragement for all those who intend to pursue a career in basic sciences or mathematics. May her memory also serve to encourage young women to break through persistent and unfounded stereotypes when it comes to young girls pursuing careers in math or science. Recognizing some of the many aspiring women in mathematics is a fitting tribute to the beautiful intellect of Dr. Mirzakhani and this is what is being done together by the AWM and AMS alluded to earlier in this article. In fact May 12^{th} the birth anniversary of Maryam will be recognised as the International day for Women in Mathematics according to an AWM announcement.

Marayam Mirzakhani *Photo credits AMS.org*

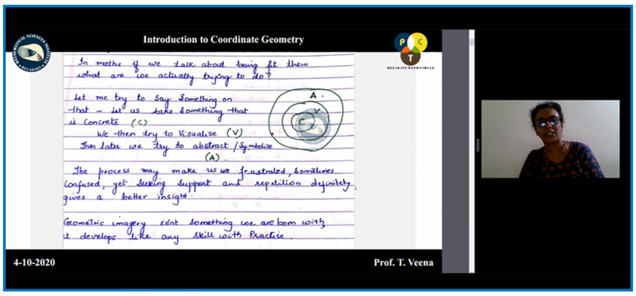
Milestones

May 12, 1977 - Birthday of Maryam Mirzakhani
1994 - First Math Olympiad Medal
1995 - Second Math Olympiad Medal
1999 - Bachelor's Degree from Sharif University Tehran
2003 - Harvard Junior Research Fellowship awarded
2004 - PhD - awarded from Harvard
2009 - Blumenthal Award winner
2013 - AMS - Satter Prize
2013 - Simon's Investigator Award
2014 - Clay Research Award
2014 - Fields Medal Award
2015 - Elected as Foreign Associate of French Academy of Sciences
2015 - Elected to American Philosophical Society.

MSI Roundup



Screen Shot of MSI Webinar MSI File Photo



Screen Shot of MSI Webinar MSI File Photo

In the month of May Mathematical Sciences Institute Belagavi was among the forerunners who took the plunge in the virtual space to have some constructive academic activity inspite of the pandemic that created a havoc in our day-to-day routine.

From May 2nd through the entire month MSI, Belagavi organised a virtual workshop for students aspiring to become professional teachers. Lectures were delivered in various topics like Linear Algebra and Matrix theory, Real and complex analysis, Algebra and topology for students aspiring to attempt the NET and SLET examinations held every a year for those who want eligibility for JRF (Junior Research Fellowships) and lecturership. Chidanand Badiger, member of MSIB gave lectures on Analysis and topology. Mallikarjun Kuri of Morari Desai Residential PU college, Udupi lectured on Linear Algebra. J. V. Ramana Raju of Jain University (also Hon-editor, MSI-Newsletter) lectured on various aspects of Matrix theory. Professor T. Venkatesh, Director, MSI gave lectures on Algebra. The event had a participants count of 38 students.

During July 26-27, MSI organised a state level webinar on Number Theory and Computing. Topics discussed were Primality, Riemann zeta function and computations involving algebraic curves.

Professor T. Venkatesh, Director, MSIB participated in the National Webinar on ‘Probability Models and Applications’ by delivering the key-note address. The event was hosted by the School of Sciences, Jain University Bangalore. Very Recently (November 3rd and 4th) MSIB organised a National webinar on Computability and Machine Learning. The themes discussed were computational complexity, P versus NP problem, Machine learning fundamentals and Bayesian statistical models. Around 60 participants that included teaching faculty, researchers and undergraduate students participated in the event.

The outreach wing of MSIB inaugurated its PU-Math Lecture series aimed at pre-university (class 11-12) students who have opted for mathematics. The said event comprises of weekend lectures in basic themes like Algebra, Calculus, Trigonometry, Number-Theory, Combinatorics and Geometry and these lectures that started in the month of October will be continued through 2021 also.

“The difference between the poet and the mathematician is that the poet tries to get his head into the heavens while the mathematician tries to get the heavens into his head.”

– G.K. Chesterton

Classics

*The classics section is devoted to an important work which is in a sense ground breaking, or an award lecture that describes such a feat. We look at the classical work of the **Abel Laureate Louis Nirenberg** who passed away early this year. The work we focus here is one of the seminal works by Nirenberg namely the Weyl problem and related embedding problems.*

Some abstractly defined manifolds cannot be intuitively imagined in the sense of submanifolds of some euclidean or projective spaces. For instance the Lobatchevskian plane is one such "surface". The Grassmanian manifold is another such example. Topologists and geometers wanted to know if such abstractly defined manifolds can be realized as submanifolds. In the words of John Nash (The Beautiful Mind-fame), one wants to know as to what extent are the abstract Riemannian manifolds a more general family then the sub-manifolds of euclidean spaces. A proper formulation of such embedding problems was done way back in 1983 by Schlaeflie. Hilbert gave a negative answer for the particular case of the Lobatchevsky plane, that it cannot be realized as a sub-manifold of $\mathbb{R}^3$. S Chern in collaboration with Kuipen gave another negative result namely for the flat-n torus that it cannot be realized as a submanifold in less than 2n dimensions. Now for the positive result, local results were available by the work of M. Janat, Elie Cartan. The concrete problem given to Nirenberg by his advisor James Stoker is whether a sphere equipped with an arbitrary metric curvature, can be realized as a submanifold of $\mathbb{R}^3$. This is precisely the Weyl problem and the answer given by Nirenberg is in the affirmative. The methods adopted by Nirenberg, established him as one of the foremost geometric-analysts while another contemporary work (that of Pogoelov) treats this question by polyhedral approximations. Nirenberg adopted the route of partial differential equations and he was led to what are now called Monge- Ampere type equations. With this reasoning backed by the theory of non-linear elliptic PDE's, the existence of such an embedding was proved with provision that the metric g1- given is at least 4 times differentiable. Basically Nirenberg proceeds to show that such a Riemannian metric can be a joined to the standard metric by a continuous path in the space of all metrics with positive curvature thus giving a functional analytic touch.

For the historically bent reader we would like to mention that it was Hassen Whitney who was credited to be the foremost in giving an embedding theorem. Existence of local embeddings were already conjectured by Schaefli as noted. He in fact formulated the problem as that of solving a system of PDE's. Elie-Cartan proved the existence of local embeddings for any smooth n-manifold such that the embedding is isometric. Hermann Weyl a contemperory of Elie Cartan was actually interested in global embeddings. It is in this context that Nirenberg's solution becomes important. As we know the more general embedding problem is true due to the works of John Nash and later works of J.Moser. Nirenberg in his PhD thesis solved the Weyl problem and the related Minkowiski problem also.

L. Nirenberg *Photo credits AMS.org*

Tributes to the great mathematician and institution builder C. S. Seshadri

C. S. Seshadri & Rajan *Photo credits The Wire*

C. S. Seshadri an eminent mathematician one of the torch bearers of modern algebraic geometry community of India and educationist par excellence passed away on July 17 2020, at the age of 88. Born in the leap year 1932, on February 29.

A connoisseur of Indian classical music and also recipient of Padmabhushan and a fellow of Royal Society of London, Prof. C. S. Seshadri, has many a times made India proud.

Seshadri was born into a lawyers family at a village near Kanchipuram Tamil Nadu, India. He was the eldest among ten siblings and incidentally, the youngest C. S. Rajan is also a renowned mathematician. Seshadri has left an indelible mark through his stints at TIFR, Paris, IMSc, Chennai and at the institute he founded-the Chennai Mathematical Institute.

After his initial studies at Loyala College Chennai, he went on to pursue research in mathematics at TIFR Mumbai. Along with illustrious colleagues like M. S. Raghunathan, M. S. Narasimhan and mentor K. Chandrashekeran, Seshadri helped establish the School of Mathematics at TIFR as a world class center. During the stint, he was sent to Paris (Ecole's Normale Superieure) on deputation between 1957-60 where he imbibed the contemporary mathematics and also the culture of the land influenced by the brilliance of Greats like Claude Chevalley, Elie Cartan, Laurant Schwartz, Alexander Grothendeick and J. P. Serre. Seshadri returned with a renewed confidence to establish a modern group of algebraic geometers in India. During this period Seshadri successfully worked on the Serre's conjecture and solved its first non-trivial case.

The Serre's problem is to establish "freeness" of a certain projective module. If P is a projective module over the ring of polynomials with n-indeterminates(over any field), then one has to show that P is a free module. The problem envisaged by John Pierre Serre had been backed by certain observations in the language of projective geometry.The language of algebra has connections to vector bundles on suitable topological spaces a theme, which appears in Seshadri's later work. The equivalent geometric statement of his pure algebraic work is that the vector bundles on the affine plane are trivial.

During his stay at Paris, C. S. Seshadri proved the freeness of Projective modules defined over k[x,y] the polynomial rings over two variables over any field k. The complete solution(i.e for any number of variables) had to wait till 1976 when D. Quillen and A. A. Suslin settled the problem independently but almost at the same time.

Later teaming up with his friend and colleague M. S. Narasimhan, they worked on the moduli of vector bundles on Riemann surfaces. They could relate stable vector bundles on compact Riemann surfaces to unitary representations of the fundamental group of the respective surfaces. This led to the construction of moduli spaces of vector bundles for projective algebraic curves.

The Narasimhan-Seshadri theorem(1965) summarizing

the above said work is one of the most cited works in algebraic geometry. In his later years he began working on new constructs like the standard monomial theory (together with Lakshmibai and C.Musili) and parabolic vector bundles. Seshadri's administrative capabilities blossomed when he took up the charge of developing the SPIC Mathematics Institute which later got christened as Chennai Mathematical Institute (Deemed to-be University). Together with another music lover and entrepreneur ℝ. Thyagarajan, Seshadri could gather liked minded fine mathematicians computer scientists and physicists to set up one of the world best theoretical center devoted to developing students right from undergraduate training.
Seshadri's legacy as a first-rate mathematician and enterprising spirit has put the little institute at Chennai on the global map. He was awarded several awards, including the Shantiswaroop Bhatnagar award, the Padmabhushan award, the third World Academy of Science award, Fellowship of AMS and he also became the fellow of the Royal Society of London (FRS).

Tributes to the Mathe-magician S.S.Shrikande

S. S. Shrikhande a famous Mathematician who briefly worked at the UNC at Chapel Hill, died on 21st april 2020 at the age of 102. The editorial team at MSI has prepared this obituary article recollecting some of his landmark achievements.

His mentor was R. C. Bose the other famous personality from Indian known for theory of designs and the BHC codes. He came out from a very poor condition, since he was one of 9 siblings however, he was determined to take up a scholarly position. His father worked at a flour mill at Sagar, Madhya Pradesh. After securing a gold medal at his B.Sc Honours course at Government College of Science (Nagpur) he started working as an assistant at Indian statistical Institute Kolkata. It is here that he met his mentor R. C. Bose with whom he collaborated, while doing his PhD at UNC, Chapel Hill. R. C. Bose was in fact his research supervisor.

S.S Shrikande *Photo credits The Wire*

Now we shall come to the famous works we alluded to earlier. In the paper 'Investigations on a new type of magic squares', Euler discusses Greco-Latin Squares(the name magic squares is a misnomer in the said article). Latin square as we know is a sudoku type n-square with each number between 1 to n appearing only Greco Latin squares instead have ordered pairs (x,y) as enteries, such that x belongs to first n latin letters and y belongs to first n Greek letters.

Euler conjectured that it is impossible to construct Greco-Latin squared of orders 4n+2. Considering the period of Leonherd Euler, one infers that this was a long standing conjecture(about 177 years old). Finally in 1959, Shrikande, R. C. Bose and E T Parker announced a proof for the existence of Greco-Latin Squares of orders 10,14,18 and so on. They also gave precise constructions for some concrete cases. The trio earned the "Euler's Spoilers" as it turned out to be a New York Times sunday headlines.

Living Legends

Peter Sarnak - IAS, Princeton   Photo credits The Wire

In this sections we try to explain in brief the work of some internationally acclaimed mathematicians who are active currently working on some important problems in the mathematical sciences and allied areas.

First up we have picked up the case of Professor Peter Sarnak whom one of the authors in the editorial desk at MSI interacted nearly two decades ago. Professor Peter Sarnak of the Institute for advanced study, Princeton (IAS), is one of the leading analysts and number theory specialists in the contemporary times. Born in 1953 Professor Peter Sarnak is a South-African born Mathematician (born in Johannesburg). After obtaining a B.Sc. degree from the University of Witwatersrand in 1975, he moved to California to continue his studies. He has been Eugene Higgins Professor of Mathematics at Princeton University since 2002, succeeding Andrew Wiles, and is an editor of the Annals of Mathematics, one of the most prestigious journals in mathematics along with his permanent faculty position at the IAS. He is known for his work on cusp forms that led to the negation of a conjecture of Atle Selberg His work has distinctly shaped analytic number theory by introducing new and powerful spectral-theoretic techniques to the study of many deep and longstanding problems. One of Sarnak's most outstanding contributions to number theory is his study of zeros of the zeta and related L-functions. After receiving his PhD from Stanford University in 1980 under the direction of Paul Cohen, Professor Sarnak was head at the Courant Institute of Mathematics, NYU. He is noted for his groundbreaking work that cut across disciplines combing sometimes number theory and analysis and some other times combinatorics and analysis and so on. His latest work on a special kind of expander graphs known as Ramanujan graphs involving sparse families of graphs gained a great deal of international attention. He is one of the foremost mathematicians who collaborated to give an explicit construction of Expander graphs much needed by the computer science community. Some awards and accolades include the Frank-Nelson Cole prize of AMS, the SIAM-Polya prize, the Ostrowski prize, the Levi-Conant Prize, the Wolf-prize and the Sylvester medal. He was elected as member , National Academy of Sciences of the United States in the year 2002. He is also a Fellow of the Royal Society of London. Here are his humble words: "as far as the profession are concerned I enjoy teaching, and I very much enjoy having graduate students I've had very many graduate students, and often I learnt from them maybe more than they learnt from me." His insights and "readiness to share ideas" have been profoundly inspirational for the work of students and fellow researchers in many fields of mathematics. Peter Sarnak has supervised a remarkable number of PhD students (nearly 50) and many of them have become well-established prominent mathematicians at first-rate universities world-wide.

Mathematics is as much an aspect of culture as it is a collection of algorithms. — Carl Boyer

Ingrid Daubechies *Photo credits Simon Foundation*

Ingrid Daubechies is our second pick among the living legends. Daubechies is recognized for her study of the mathematical methods that enhance image-compression technology. She is a member of the National Academy of Engineering. Often in Mathematical circles and communities, the applicability aspect becomes secondary importance given to pure mathematical thoughts and aesthetics associated. But in case of the recognition given to Daubechies. we can say in several counts that she has made a difference to the norms in the mathematical arena. She has truly developed the notion of mathematics as a technology. The name is widely associated with the orthogonal wavelet technique and the associated biorthogonal families of basis functions. A wavelet from this family of wavelets is now used in the JPEG 2000 standard. JPEG stands for Joint Photographic Encoding Group which creates standards for image compression and coding systems. We shall henceforth refer to her as IG. Automations in the field of multimedia is mainly due to the technologies developed by IG and she also developed sophisticated image processing techniques used to help establish the authenticity and age of some of the world's most famous works of art including paintings by Vincent van Gogh and Rembrandt. A Belgian by birth IG completed her undergraduate studies in physics at the Vrige Universiteit.

Then she completed graduate studies working in Quantum Mechanics at CNRS Center for Theoretical Physics and the Free University in Brussels.

In 1987 IG joined the Murray Hill-AT & T Bell labs, New Jersey facility and it is during this time her work on orthonormal bases of compactly supported wavelets got published in the journal, Communications On Pure and Applied Mathematics. She soon became famous among math-communities and IG began teaching as a Professor at Rutgers University in their Mathematics Department. IG then moved to Princeton University where she was active within the Program in Applied and Computational Mathematics. She is currently with the Department of Mathematics and electrical and computer engineering at Duke University. IG is credited to be the first woman president of the International Mathematical Union.

Daubechies received the Louis Empain Prize for Physics in 1984. In January 2005, IG became the third woman since 1924 to give the JW Gibbs memorial lecture at AMS. She was a plenary speaker at the ICM-Zurich 1994. Besides IG won the William Benter Prize in applied Mathematics. Another famous award IG won was the Leroy P. Steele Prize for seminal contribution to research for her contributions to Orthonormal bases of compactly supported wavelets. Part of the citation says .... The orthonormality makes them good as a basis to decompose arbitrary signals; the smoothness removes edge artifacts and makes wavelet series converge rapidly; and the compact support makes them viable for use in actual practical applications. The wavelets also came with a parameter that traded off their smoothness for the width of their support and amount of oscillation, making them flexible enough to be used in a variety of situations. As such, these wavelets (now known as Daubechies wavelets) became extremely popular in practical signal processing (for instance, they are used in the JPEG 2000 image com-pression scheme). Even nowadays they are still the default, general-purpose wavelet family of choice to implement in any signal processing algorithm (although for specialized applications, sometimes a more tailored wavelet can be slightly superior).

Interesting Lives, dedicated to algorithms and coding

It is very rare to see women working at the pinnacle level for technological giants like AT&T Bell Labs, Microsoft. In this feature article we describe a couple of personalities whose interests originate in pure mathematics but at the same time their work impacts technology.

Florence Jessie MacWilliams (1917-1990) whom we refer to 'Jessie', was a very determined and hard working mathematician. Though she started her research work under the famous algebraist Oscar Zarisi at the age of 22, she completed her PhD only at the age of 44, as she took a break to raise her three kids. Since her engineer husband was already working at AT&T Bell Labs, Jessie joined there as a computer programmar, but soon she became ambitious to take up more challenging roles on the research community there. She returned back to Harvard during the 1960's and wrote a remarkable thesis entitled "combinatorial problems of elementary group theory". Interestingly her daughter Anne Macwilliams was also a graduate student at the Harvard maths department during this whole saga.

In fact it was through her daughter that Jessie got her new PhD advisor Andrew Gleason. As we know, in the digital communications environment, the error correcting codes play an important role, and the thesis problem of Jessie contained important combinatorial results concerning coding and decoding algorithms. Jessie in fact gave a formula that related weight distribution of a linear code to that of its dual code. A related result named after her is the generalized Macwilliams function for certain error-correcting coding schemes. Jessie's later work concerns other types of codes that involve cyclic groups and matrix designs. Interestingly Jessie is also a co-author with Fields medalist J. Thompson. Teaming up with N.J.A Sloane she wrote the encyclopedic book "The Theory of Error Correcting Codes", (1977, North Holland Publications). In 1983, when the Association for Women in Mathematics (AWM) inaugurated its lecture series in the memory of Emmy Noether, Jessie gave the first Emmy Noether lecture.

Computational number theory and related geometric objects are close to the heart of **Kristin Lauter**, whom we feature as the second interesting personality, currently the principal researcher and research manager of the cryptography group at Microsoft research in Redmond Washington. Kristin is particularly popular for her research on "homomorphic encryption" a term associated with private A.I. When machine learning experts works on some of their clients data, we have this doubt that our data may be stolen. But "private - A.I" concept helps us allow ML experts to work on encrypted data and yet give required conclusions (which are again in encrypted format). Kristin received her PhD from the University of Chicago in 1996 under the supervision of Niels Ovesen Nygaard.

Prior to joining Microsoft, Kristin was an assistant professor at University of Michigan, and she also held some research positions at the Max Planck Institute Germany and Institut de Mathematiques Luminy, France.

Besides, Kristin was elected President of the Association for Women in Mathematics(AWM) for the period 2105-2019, she was the George Polya Lecturer at MAA recently, and she is the co-founder of the Women In Numbers Network(WINN). She is part of class of 2015 fellows of the AMS. One can read her beautifully written article in the 'medium' an contemporary Cryptography [https://medium.com/create-the-future/-70d5a70a3536]. The Society for Industrial And Applied Mathematics(SIAM) awarded their prestigious fellowship for the development of practical cryptography and for leadership in the mathematical community to Kristin.

Kristin Lauter   Photo Credits AMS Notices

The MSI Samplers

Prof C. S. Rajan - TIFR, Mumbai *MSIB File Photo*

In the sampler section we generally highlight some recent expositions especially those that happen on Indian soil. We chronicle the beautiful lecture by C.S.Rajan that happened at our own Institute the Mathematical Sciences Institute Belagavi.

He recalled historical facts that the Pythagorean time philosophers and the Sumerians knew sums of squares for right angled triangles. He then described Diophantine equations and gave the example of Brahmagupta's work regarding some algorithms that he developed for generating solutions on the so-called Pell equation. On the way to explain various developments to solve polynomial equations with integer coefficients the speaker gave historical accounts of the contributions of L.Euler, Bernoulli and Kummer with regard to number theory.

In the lecture entitled Arithmetic, Prof Rajan starts off with elementary number theory themes like Primality, Sum of squares, Pell Equations, Congruent Numbers, Quadratic reciprocity etc. He detailed on a proof of Fermat regarding the statement that whenever a prime p=1(mod4) is given it can always be written as a sum of two squares. This is being explained using the method of descent by the so called reverse engineering technique. This lecture was truly a historically motivated exposition as he narrated that Euler takes up the thread of thought of Fermat. These starting points lead to a coarse quadratic reciprocity statement through the minds of Euler, Legendre and finally by Gauss.

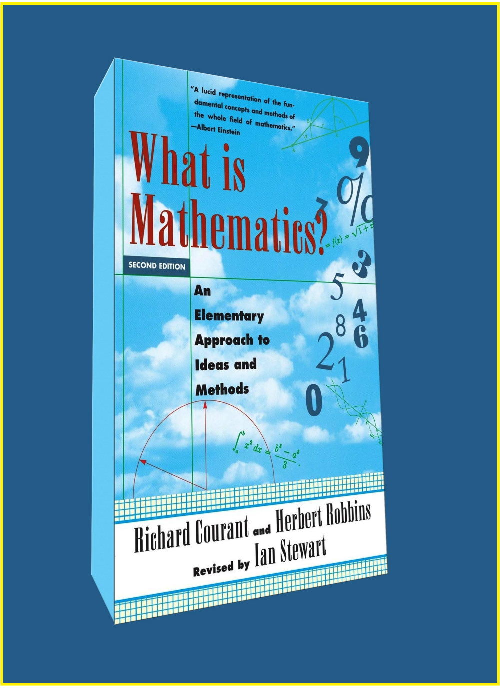
The most general statement is called the Gauss - Quadratic reciprocity law. Fix a square for integer n. Then the set of all primes p co-prime to 4n for which n is a square modulo 'p' are given by a collection of congruence classes modulo 4n. The speaker went on further to say that the reciprocity laws here, tell us precisely which are the congruence classes modulo 4n that are involved. Hasse principle a corner stone of arithmetic algebraic geometry was also introduced.

Prof T. Venkatesh-Director, MSIB - Addressing the gathering

This exposition had other related ingredients but for the sake of focus, we have described mainly the Diophantine equations and related results. Just to enlist them, Prof. C. S. Rajan also introduced the p-adic analysis to discuss local-global principles(Legendre) and Hensel's lemma.

Finally the speaker also gave glimpses of the use of p-adic numbers in relation to the analysis of the famous Riemann zeta function.

Book Reviews



TITLE: What is Mathematics?
An Elementary Approach to ideas & Methods

-by Richard Courant & Herbert Robbins

Reviewed by
T. Veena,
Head, Department of Applied Science & Humanities
V. P. Polytechnic, Belagavi

The book 'What is Mathematics' by Richard Courant and Herbert Robbins published by oxford University Press is a must read for teacher and students.

I happened to stumble on this book recently and fell in love with it assuming minimum background on the part of the reader and keeping in view the curious interests of the students the chapters in the book are well articulated.

Since a very long time mathematics is always regarded as a vital aspect of human intellect. This book covers many mathematical topics that a student encounters for the first time and the way they have to be learnt is clearly evident.

The book included chapters on natural numbers, geometry constructions, calculus, topology, differential equations and much more. The beauty of this book is that the chapters are mostly independent of each other which allows the reader to pick and choose his/her areas of interest.

The best mathematics is like poetry and music. It brings a story to life before your eyes and involved you in it intellectually and emotionally "What is Mathematics" is like a fine piece of art.

The book definitely needs a place in the personal library of every teacher teaching mathematics in schools and colleges.

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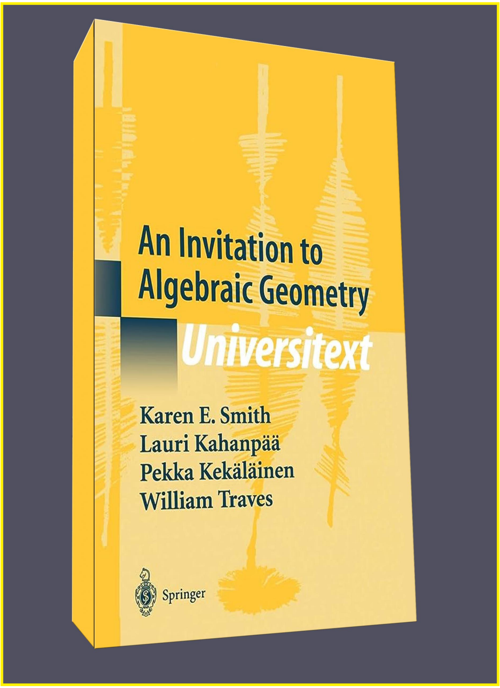
- Chapter - I: The Theory of Numbers
- Chapter - II: The Number System of Mathematics
- Chapter - III: Geometrical Constructions - The Algebra of Number Fields
- Chapter - IV: Projective Geometry. Axioms, Non-Euclidean Geometries
- Chapter - V: Topology
- Chapter - VI: Functions and Limits
- Chapter - VII: Maxima and Minima
- Chapter - VIII: The Calculus
- Chapter - IX: Recent Developments

"The art of doing Mathematics consists of finding that special case which contains all the germs of generality."

– Deepak Chopra



TITLE: An Invitation to Algebraic Geometry

by Smith Karen, Lauri Kahanpaa, Kekalainen Pekka, William Trave

Reviewed by
J. V. Ramana Raju,
School of Sciences,
Jain University, Bengaluru

While there are standard books for subjects like Real Analysis or Number theory to be used as a text book for introducing students with the rudiments of the classics, when it comes to a subject like algebraic geometry there is no one such standard, but a whole mountain range of classical books that introduce the subject to eager students. For instance the Red book of Varieties and schemes by David Mumford is used in parallel with Mumford's other book namely Complex Projective Varieties (Algebraic Geometry-I). So a book that gives a wholesome view is the one that is under review currently. This book intended for a first course is reader-friendly and well organized, with plenty of exercises, accessible to even undergraduates, especially considering the fact that this subject is quite abstract and difficult to grasp in totality. Personal experiences indicate that this is a book which makes one fall in love with the subject and ask for more. According to the preface the book is intended for the working or aspiring mathematician who is unfamilar with algebraic geometry but wishes to gain an appreciation to its foundations and its goals with a minimum prerequisites. It can be read with a sense of pleasure aided with intuition without going into too much of technical baggage. For instance the authors give us an idea of how scheme theory generalizes a geometric concept, without going into detail. Anyone with a background in basic topology, real and complex analysis and elementary ring theory could pick up this book. At the same time the authors have wisely indicated the historical high points like the classical work of the Zariski school, and Grothendieck's modern formulations.

The book starts off with a chapter on affine algebraic varieties and gives a concise introduction through plenty of examples. This is followed by the algebraic foundations essentially comprising of the much needed commutative algebra and constructions therein. The chapter on projective varieties appeals to the geometrically inclined reader. The next chapter on classical constructions leads us into more abstract algebraic geometry, for instance the Hilbert polynomial method is dealt in great detail. The remaining chapters are focussed to the analyst who would want to understand concepts like smoothness and sheaf theoretic perspectives for the local theory and maps between varieties to look for classification themes. Overall this book is an interesting read for anyone wanting to get a quick flavour of the subject for a one-semester course.

Interestingly there is a French translation and a Persian translation available.

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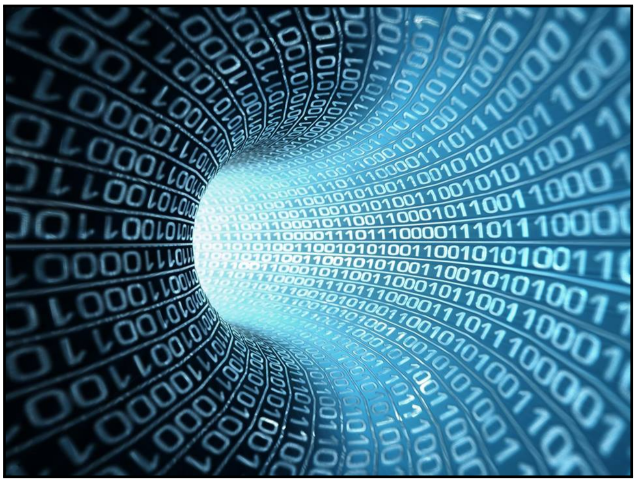
- Chapter - VIII: Maps to Projective Space

Math-Education

This space is meant for math-teachers to discuss issues related to pedagogy. Often we take up issues and their discussion that are commonly raised in Math-forums worldwide.

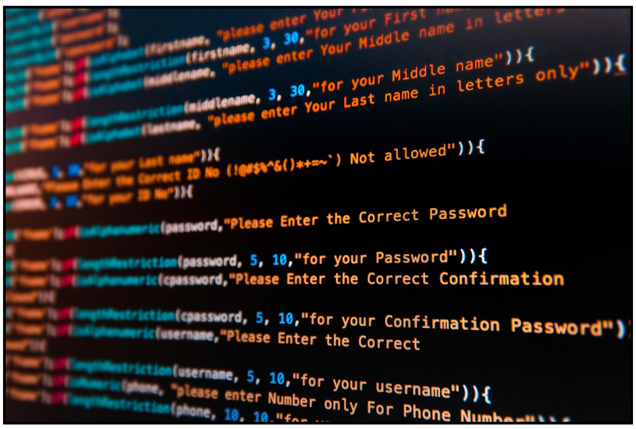


Technology as an engine to accelerate pedagogy - A discussion

When we think about technology one thing that comes to mind immediately is teaching/learning mathematics using the World Wide Web or computer. The second aspect is the use of symbolic computations (like SAGE, GAP, SCILAB, MATLAB etc). We also discuss in this article the various aspects concerning cognitive abilities especially for the slow learners difficulties related to understanding specific topics could be overcome by the technological advances.

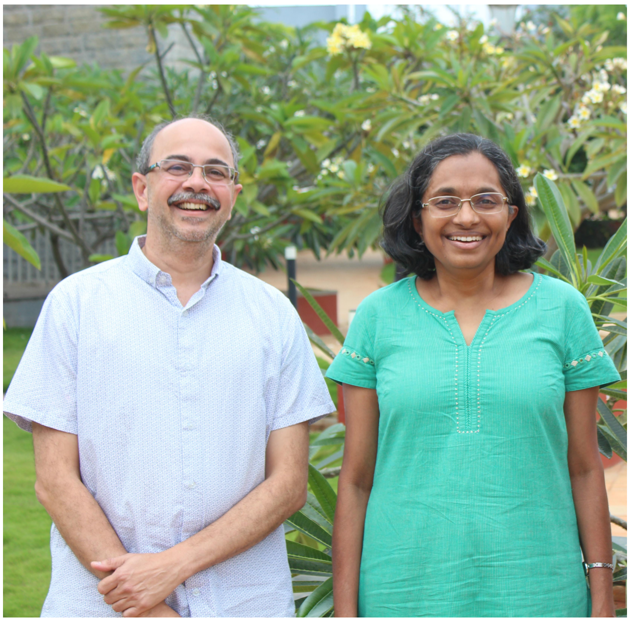
The availability and use of the World Wide Web are proliferating quite rapidly, even in developing countries like India. MIT has been giving several resources resources free for online browsing and downloads. Some of its ocw (open courseware) includes multimedia content like videos, and slides also. Interactive tools like google-classrooms, slideshare can aid the teachers community provided the instructors make a blend of their own instruction and online resources. The delivery of online courses will almost certainly continue to get more sophisticated and more interactive, something that is especially good for self-directed learners and those who need "just in time" information. This, together with automated question answering and ollow-up suggestions, provides a high degree of interactivity. There are also computer algebra systems (CAS) and other mathematical software for computers, as well as various graphing and other calculators. Are we making better use of CAS and other software? For instance this author has been using animations to help learners understand actions of matrices and polynomials in certain cases. While traditionally many of the uses of the computer are used to be computational, enabling students to investigate problems involving "messy" real world data, others like the ones mentioned above are meant to facilitate both procedural and conceptual learning of mathematical topics. For the students needing a small push to visualise and appreciate math-concepts technology can definitely aid.

Coming back to online courses from the perspective of teachers while it is generally conceded that development of such multimedia material and course-slides takes much more instructor time, they are not as ephemeral as traditional classroom-delivered university courses because they remain on the Web for some time. So the really slow learners can take their own time to re-run the multimedia resources and get the required subject knowledge. But there is a downside that we are observing. When such open resources are suggested, students often tend to pick concepts in a piecemeal fashion just to satisfy their examination needs.



The Y-Space *Youtubers and the like*

In this issue we have started this new series, where we identify some really nice video treats for Math-enthusiasts. This time we are covering International Centre for Theoretical Sciences. Youtube channel named "International Centre for Theoretical Sciences. Youtube channel".



Prof. Amit Apte and Prof Rama Govindarajan both of ICTS, Bengaluru

Again in this we will look into the play-list concerning the women in science programmes.

Since we are celebrating women in Mathematics. Here we find an interesting list that we suggest. One of the speakers here is Dr.Rohini Godbole, she chronicled several women mathematicians.

Dr. Rama Govindrajan gave a beautiful exposition of fluid mechanics, including that of the mechanics of raindrops. Dr. Manjunath Krishnapur gives a fabulous talk on problem solving and related theory building. Dr. P.P. Divakarana gives a historical account of Madhava's contributions to calculus.

Also in the applied fields we have an amazing description of math- modelling for chemical reactors by Dr Preeti Aghalyam.

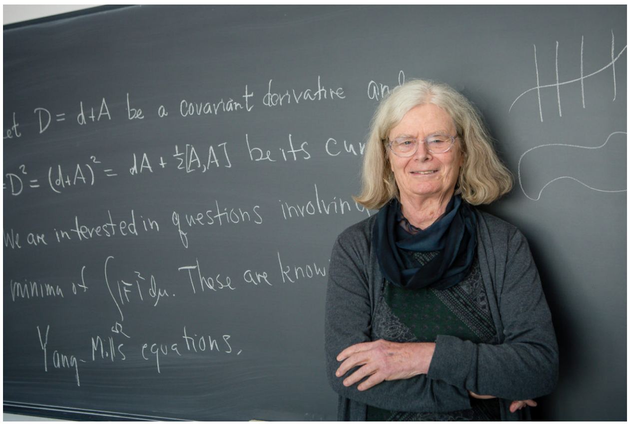
Somewhere between pure math and pure applications we have art form inspired by math. Mr. VSS Sastry describes the origami aspects while discussing mathematics.

Tanvi Jain spoke on the mysteries associated with the napier base 'e'. She took up simple themes like limits, sequences etc to drive home the point about the importance of this irrational number Nutan Limaya of IITB spoke on theoretical computer science namely how is efficient computation done. In an earlier version of the women in math conference she had a fancy title: hats off to theoretical computer science.

Participants discussing during the workshop

Participants discussing during the workshop

Around The World....The browsing fingers!!



Karen Uhlenbeck *Photo Courtesy AMS-Notices*

In October of 2020 MIT PRIMES, founded by Pavel Etingof and Slava Gerovitch, celebrated its tenth anniversary. It all began as an experiment in the year-long math research by high school students with just around twenty-one local participants. The experiment has been proven very successful, with the program growing more than five-fold in ten years and expanding both nationally and internationally(one can also read the praises and accolades for this innovative programme).

PRIMES is actually a free, year-long research and enrichment program for high school students, created in the MIT Mathematics Department in October 2010. The program's innovative, year-long model for guiding high school student research is being used also in computer science, bioinformatics, computational and physical biology, genomics, and neuroscience. PRIMES students use their knowledge of mathematics and computer programming to crack problems related to cancer research, Internet security, traffic control, refugee migration, brain research, laser engineering, and many other real world fields. PRIMES Circle and Math ROOTS the two offshoots of this programme help attract members of under represented groups to pursue careers in the STEM fields. A very interesting prize that is instituted by AMS (the American Math society) is the Award for an Exemplary Program or Achievement in a Mathematics Department. This was established by the AMS Council in 2004 and was given for the first time in 2006 to the Department of Mathematics at Harvey Mudd College. This years recipient is the PRIMES programme of MIT.

Let us explore what this is all about. PRIMES offers real, not toy, research projects to high school students and provides academic mentorship for a full year. The program builds collaborative teams that in-clude faculty, postdoctoral researchers, graduate students, undergraduates, and high school students, promoting part-nership and wider outreach in the mathematical sciences community. Keys to the PRIMES success are thorough preparation, continuous review of research projects, and effective men-torship techniques. Choosing a research project for a high school student is no easy task. PRIMES's experience shows that most fruitful are the projects that have an accessible beginning with relatively simple initial steps; flexibility in switching among several related questions; computer- assisted exploration aimed at finding patterns and making conjectures; faculty advisor involvement; relation to the mentor's own research area; understanding of the big picture and motivation; a learning component that encourages the student to study advanced mathematics; and do ability within a year-long time frame. Effective mentorship at this place involves striking a balance between guiding the student and allowing independent thinking, being attuned to the learning and research style of every student, and regularly reviewing the project progress and adjusting its scope, if needed. Head mentor Tanya Khovanova regularly meets with students to gather their feedback and help with communication and motivation issues.

CITATION The 2020 Award for an Exemplary Program or Achievement in a Mathematics Department is presented to the Department of Mathematics at the Massachusetts Institute of Technology. The MIT Mathematics Department is being honored for its

Program for Research in Mathematics, Engineering, and Science for High School Students (PRIMES), which provides significant research experiences and mathematics enrichment to high school students locally and globally, with particular attention to increasing the representation of women and under represented minorities.
Here is a testimonial of an award winner at PRIMES "PRIMES is an incredible opportunity that allows high schoolers to do what they would never normally have the chance to do: research, while also providing the guidance and encouragement that is crucial for success. Ultimately, PRIMES has truly cemented my interest in math, and it is for this reason that I would definitely encourage any student similarly passionate about mathematics to apply!".

The Joan Birman Fellowship (News-courtesy AMS-Notices) The Joan and Joseph Birman Fellowship for Women Scholars is a middle level career research fellowship specially meant to fit the unique needs of women-mathematicians. Open only to women, these fellowships program are made possible by a generous gift from Joan and Joseph Birman. The awards started being given from the year 2017. The fellowship seeks to address the paucity of women at the highest levels of research in mathematics by giving exceptionally talented women extra research support during their mid-career years. The most likely awardee is a mid-career woman, based at a US academic institution, with a well established research record in a core area of mathematics. The fellowship will be directed toward those for whom the award will make a real difference in the development of their research career. Candidates must have a carefully thought-through research plan for the fellowship period. Special circumstances (such as time taken off for care of children or other family members) may be taken into consideration in making the award.
The fellowship can be used to provide additional time for research of the awardee or opportunities to work with collaborators. This may include, but is not limited to, course buyouts, travel money childcare support, or support to attend special research programs.

Abel Prize 2020 announced.

The Abel Prize 2020 was announced just after the release of our last Newsletter. Here is the update from our editorial newsroom.

Professor Gregory Margulis of the Yale University USA, and Professor Hillel Furstenberg of Hebrew University, Israel (both retired professors) share this years Abel Prize one of the prestigious awards in Mathematics. The prize committee instituted by the Norwegian government in the honor of Neils Henrik Abel will split the award money of 7.5 million Norwegian kroner, something which is slightly more than $700,000.
The citation for the prize, awarded by the Norwegian Academy of Science and Letters, lauds the two mathematicians "for pioneering the use of methods from probability and dynamics in group theory, number theory and combinatorics". Previous laureates include Andrew J. Wiles, Peter Lax, John Nash and Luois Nirenberg who is featured in the classics section of this issue. Prof Karen Uhlenbeck Emeritus professor at the University of Texas at Austin last year became the first woman to become an Abel Laureate.

Karen Uhlenbeck incidentally was the first women to receive the Abel Prize in Mathematics.

"Did you know that every square is a rhombus, but not every rhombus is a square? I mean, you can't make this stuff up!"

Math in the Media

In this section we describe some illuminating articles appearing in the media (general or science related newspapers/magazines) which give us a wider view of what's happening in the mathematical sciences

Prof. Cedric Vilani (Centre) walks along the roads of Paris

Nature Magazine described the trefoil knot while discussing various aspects of molecular dynamics. Algebraic topology terms were described in several places of the article.

In an NYT (New York Times) article mathematical models have been prominently covered while discussing covid related predictions. In an article authored by S.Roberts the famous SIR model and its modifications have been underlined.

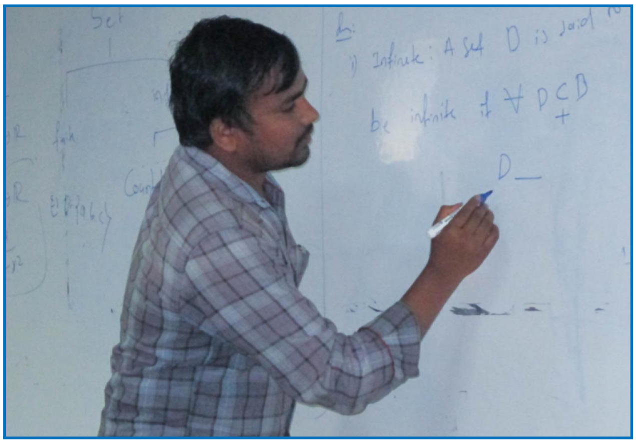
Fields medalist Cedric Vilani again featured in the NYT in a report early this year when he stood for a mayor elections at Paris. An interview of him gives glimpses of the mathematician discussing the application of some optimal transport problems for the betterment of the society. Traffic problems and related mathematical and A.I algorithms were also mentioned. In the news- website "The Wire", Atanu Biswas a Professor at ISI-Kolkota writes about the mathematical features (game-theory) while describing the work of this years Nobel laureate in Economics. Various optimization schemes coming up in the Milgram-Wilson's auction designs have been discussed in the said article. The website "Firstpost" described among other things, the Shantiswarup Bhatnagar award given this year to the mathematician U K Anandavardhanan for his work in abstract group theory and related representations.

The sad demise of one of the leading international mathematician C. S. Seshadri was covered widely in Indian newspapers and magazines. This also became an occassion to describe and showcase the achievements of the Indian Mathematician especially in the areas of Algebra and Algebraic Geometry and also his institution building capacities.

The Frontline magazine had an article by T. Ramadas entitled "Music of Spheres". The news website scroll.in published an article written by a colleague of CSS at CMI, Prof. S. Ramanan, the article appeared with the title "From: World Class Algebraic Geometer, Institution Builder and Music Lover, India loses a pioneer in Mathematics?. The Bhavana Magazine came up with an interview of C S Seshadri "Proof to Transcendence via Theorems and Ragas"

"You learn new skills after being here awhile."

Problem corner



Chidanand Badiger, Fellow MSIB *MSIB File Photo*

Our Problem section is unique in the sense that it provides some aid for early grad students to think about some problems in such a way that along the way they also learn the theory related to the problem or a class of problems. Illuminating solutions shall be featured in subsequent issues.

Introduction: Theory of fixed points has [1,2] prominent role in the solutions of any problem in many branches of science. Brouwer fixed-point theorem, Banach fixed-point theorem, Atiyah-Bott fixed-point theorem, etc. are well-known results in the broad arena of fixed points theory. Existence results of several problems are in fact [1] consequences of some fixed point theorems.
Statement of the Problem: With the following two definitions, 1) Fixed-point property: Let $\mathbf{C}$ be a category and M be an object in this category then M is said to have fixed point property if every morphism $f : M \to M$ in the category $\mathbf{C}$ has a fixed point.

2) Topological invariant or Topological property: A property of a topological space which is invariant under homeomorphisms.

Show that fixed-point property of an object in category $\mathbf{C} = \mathbf{Top}$ is a Topological invariant.

References

[1] Agarwal, Ravi .P; Meehan, Maria; O'Regan, Donal. Fixed Point Theory and Applications, Cambridge University Press. (2001) ISBN 0-521-80250-4.

[2] Brown, R. F., Fixed Point Theory and Its Applications, American Mathematical Society (1988). ISBN 0-8218-5080-6.

Note: Readers are invited for solutions to this problem through email to MSIB (msibelgaum@gmail.com) before 30-01-2021. First come accurate and innovative solution will be published in the subsequent issue of the MSIB Newsletter including all readers' names who have sent a correct answer. Therefore, it is requested that the reader should send Name, Email, Affiliations, (references if necessary) along with their solution.

Upcoming Events

- MSI, Lectures on Algebraic Geometry For Undergrads (Tentative dates-16^{th} of November onwards, Weekend lectures)

- The traditional History of Mathematics lecture will be organised on 22^{nd} of December 2020.

- Instructional Conference on Mathematical Finance (Tentative 3^{rd} week of January 2021).

- Lecture Series on Discrete Mathematics, Random Graphs and Ramsey Theory (December 2020).

- A 3-Day Work-Shop on Complex Net Works (Tentative dates January end of February 1^{st} Week of 2021.

- Mathematical Sciences Institute Belgaum will organize a 3-Day Work Shop On Coding Theory during 2^{nd} Week March 2021.

- Mathematical Sciences Institute Belgaum will organize a 2-Day Work Shop On Lie Theory during 3^{rd} Week of March 2021.

- On 22^{nd} December the National Mathematics Day MSIB will be hosting a Math-fest for college students that includes a Poster - competition.

About the Cover Page:
The cover page is adopted from the wikimedia (website) The picture features Maryam along with a background that represents part of her works. The hyperbolic surface and its dynamics has been shown by the illustrations. The Riemann surface on the left side of the pictures is an open surface representing the class of bordered Riemann surfaces.

About the Logo

Ideas for the new logo of the Institute came in a discussion session that happened during August 2020. The image pictured at the right is inspired by the mobius band which comes up in various themes like algebraic topology, projective and differential geometry.

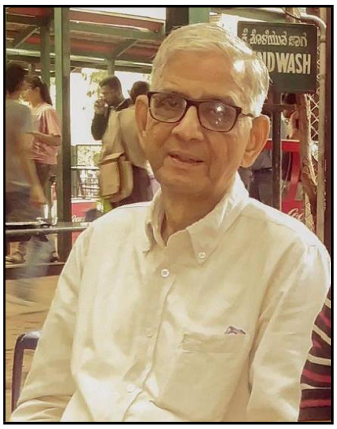
The genesis: Professor T Venkatesh, Director of the Mathematical Sciences Institute for the purpose of the new logo created a mobius band with a painted sheet of paper so that the one-sidedness of the surface could be felt. Later the same was caught on a camera and based on the same a logo was created. This was formally used as a new logo during one of the Institutes programmes in September 2020.

NEWS
MSIB has instituted Research awards for MSI Interns. Pooja B of Vidhyagiri College of Arts and Science, Karaikudi and Vidyasagar Mysoremath of RCU, Belagavi have been awarded the research fellowship for the academic year 2020-2021. The fellowships are aimed at encouraging the young and enthusiastic participants of MSI programmes.

Pooja B

Vidyasagar

FACE2FACE



Prof. K.B. Athreya, IISc, Bangalore & Mayuresh Gokhale-MSIB, Correspondent

Professor K. B. Athreya a luminary academician at Iowa State University U.S.A and Visiting Faculty IISc Bangalore visited Rani Chanamma University Belgaum and delivered a series of lectures at various institutes in the city. He spent some time at the Mathematical Science Institute Belgaum(MSIB) expressed his views on the deterioration of Pure Sciences and overall education scenario. Following are his excerpts with MSIB correspondent Mr. Mayuresh Gokhale.

Q: A recent noticeable trend in the field of higher education is the desertion of pure science courses, for want of lucrative careers in the field of I.T. Do you see this as a positive development?

A: Absolutely not. Infact I am really worried about the dwindling of traditional sciences which is a core form of knowledge.

Q: People usurping good IT skills, getting high pay packages, generating large amount of wealth and boosting the economy. Where do you find the lacuna?

A:The trend of drifting to professions like I.T, Software etc in anticipation of high paying jobs in a quick time frame has risen to unprecedented levels. Although the people in these fields earn a lot of money thereby contributing to the economic boom, the ramifications are enormous. These jobs are highly demanding and require long hours of work exceeding 14 hours a day. Moreover these jobs with no intellectual enhancement of an individual. There are instances of burnouts, where the individual faces severe psychological problems. He may not be able to enjoy life or develop any creativity leading to personal unhappiness. More importantly his life turns out to be self centered without any contribution to the society.

Q: In eighties we talked of 'brain drain' and today an enormous amount of drift to IT, but this time within the country. Are we not losing the services of bright students to innovation?

A: The question reminds me of the situation in the 50' s and 60's, when Civil Services used to be a sought after career option. Any brilliant student at that point of time dreamt to be a Collector or a Secretary etc ,which shifted enormous talent from a potential innovator to Civil services. But the quantum of drift in the IT is enormous which is taking unprecedentedly large thinking and innovative minds which would other wise contribute to our research and compete with the world in acquiring patents thereby demonstrating competitive advantages in businesses. There are instances of great dearth of skilled work force at leading National research Institutions in the country.

Q: A situation where hardly any student gets enticed towards Undergraduate Pure Sciences, Can we construe that these courses are facing Obsolescence and would it be imperative

to revamp the course contents to meet the requirements of the job market?

A: Basically the word obsolescence is a myth. The relevance of traditional sciences is persistent. In fact they are the core of knowledge that forms the basis of any technology. It is only the Institutions which are coming up with certain monotonous skill based curriculum catering to the needs of job market. But it would be incorrect to call these fundamental streams like Physics, Chemistry, Biology and Mathematics as obsolete.

Q: Given a choice to design a curriculum at undergraduate level, what are the points, On which you will emphasize?

A: The existing curriculum at undergraduate level is reasonable one. Only a dimension which could be added is that it should take note of recent trends in the respective domains and the course should be made rigorous.

Q:How would you differentiate an Undergraduate Science course in the U.S and India?

A: Essentially owing to a good faculty who are paid handsomely, the best of the experimental infrastructure and more intellectually challenging curriculum which is rigorous, have made the courses more advanced in the U.S.

Q: What according to you should be the traits of a good teacher?

A: The first thing is, he should thoroughly enjoy his profession, try and give basic concepts and anticipate things uncertain to the students, make use of good reference books eventually should be in a position to inspire his students.

Q: How would you evaluate the higher education scenario in India?

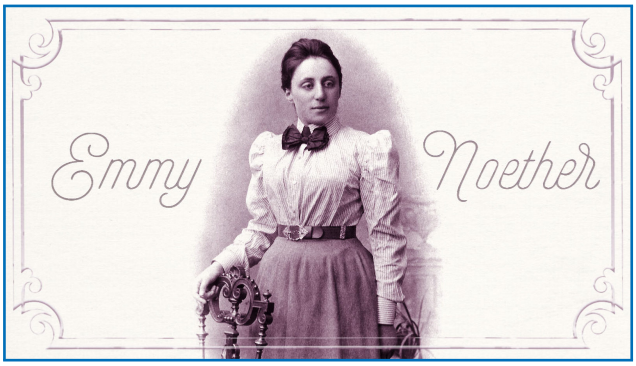
A: The teaching community should be aware of the latest trends in the areas and need to spend a bit of time in making progress by learning new things. This could be in the form of attending Conferences, Presenting Research Papers and gaining practical training. To my mind the learning process is unending. Any new piece of information should be shared with the students to create interest and build inquisitiveness.

Q: How would you relate the nexus between Higher education and development of a nation?

A: In my view a chunk of consumption oriented upper middle class are spawned by excessive IT boom. Their life styles have become self - centered with little concern for the society. A higher- education system with a human face will make the students aware of the problems prevailing in the society. The problems of poverty, basic education, health care and environment are ubiquitous in any developing nation. In a way this boom has initiated a trend of non contribution to the society which has hampered the amelioration process. Apart from this the society tends to deteriorate in large number of aspects like quality of life, intellectual effort etc which could be a big blow to any nation's growth and development.

From right - Prof K.B.Athreya, Prof T. Venkatesh, Mayuresh Gokhale, J.V. Ramana Raju

Section On Women Mathematicians



Emmy Noether Wikipedia/Artwork by Sandbox Studio, Chicago

Emmy Noether about whom a biographical article follows was a famous mathematician and a pioneer of sorts worth emulating and hence the association of women in mathematics have rightfully named their annual special lectures as Noether Lectures. The International Congress of mathematicians also has a segment meant for women in mathematics and there (since 2010) the ICM Emmy Noether special lectures are delivered.
This issue being themed at Women in Mathematics, we have collected details of 4 women mathematicians who have delivered the Noether lectures. The four women mathematicians (apart from Emmy Noether herself) we have featured here are Cathaleen Morawetz, Dusa McDuff, Karen Uhlenbeck and Ingrid Daubechies.

Emmy Noether:- Emmy Noether has been a trailblazer in the field of mathematics and mathematical physics especially because of her unique style of doing mathematics and disseminating mathematical ideas. **Amalie Emmy Noether** was born on March 23, 1882 at Erlangen Germany to Max Noether (the famous Physicist) and mother Ida Amalia Kaufmann.
As a career plan made earlier Noether got certified to teach English and French in schools for girls in 1900, but she instead chose to study mathematics at the University of Erlangen (now University of Erlangen-Nürnberg). At that time, women were only allowed to audit classes with the permission of the instructor. She spent the winter of 1903-04 auditing classes of great mathematicians of the time (at Gottingen) like David Hilbert, Hermann Minkowski, Felix Klein and Schwarzschild. She returned to Erlangen in 1904 when women were allowed to be full students there. She received a Ph.D. degree from Erlangen in 1907, with a dissertation on algebraic invariants. She remained at Erlangen, where she worked without pay on her own research and assisting her father, mathematician Max Noether.

At that time women were not given licence to teach and amidst a lot of opposition from the entire philosophical faculty, which included philosophers, historians, natural scientists and mathematicians Hilbert and Klein appointed her a privatdozent (although she was teaching under the name of Hilbert) and here she began to explore the algebraic theories to explain Albert Einstein theory of General Relativity. Here she also discovered facts from mathematical physics, as in if the Lagrangian does not change with co-ordinate system then there is a quantity that is conserved. For example, when the Lagrangian is independent of changes in time, then the energy is the conserved quantity. This relation between what are known as the symmetries of a physical system and its conservation laws is known as Noether's theorem and has proven to be a key result in theoretical physics. After this Emmy Noether got recognised as a formal academic lecturer in 1919. in collaboration with her Gottingen colleague Werner Schmeidler, she wrote and published the paper "Concerning Moduli in Noncommutative Fields, Particularly in Differential and Difference Terms" (translated from German). This made here distinctively noticeable as a scholar with extraordinary mathematical talent. She also worked extensively in commutative algebra and non-commutative algebra. Along with academic work she also used to assist editorial work for the journal *Mathematische Annalen.*

With the Nazi diktat it was time for the Jewish Professors to leave and she chose to join Bryn Mawr college in the United states. In the April of 1935 she died suddenly due to complications from an ovarian cyst.

MSI Programme on Women in Mathematics

Mathematical Sciences Institute has been regularly conducting programme for women in mathematics we had invited three women mathematicians and we are describing the lecture samplers. we also discuss here about the women in math programs being held elsewhere. In this issue we have already spoken about Emmy Noether recently *Noethember* was celeberated where caricatures covering ideas about Noether and women in mathematics in general were depicted each day one caricatures were publicized covering a month.

Prof. Arathi Sudharshan (*Area Head Department of Data Analytics and Mathematical Sciences JGI*)

Prof. Arathi Sudharshan spoke on the math-models applied in Biology, Bio-Math. She said, is an upcoming area of research where computational methods are used to figure out various complex phenomena in the human body and other life systems. Mathematics in biology are two seemingly distinct fields but they are deeply interconnected. If we see it from a modeling perspective especially in the realm of cardiology the study of heart and its function has benefitted largely by the applications of mathematical modeling. Such studies unravel complex interactions withing biological systems.

Numerical alogorithms form a particular base for analysing various phenomena. Take for instance cardiac geometry data (ECG). In particular she spoke about the aspects in physiology of heart where spatially distributed physical systems are studied by a model using analogy of electric circuits.

Anisa Chorwadwala (*Associate Professor IISER Pune*)

Prof. Anisha in her talk raised some nice questions concerning geometric analysis. Why are soap bubbles that float in air, approximately spherical or why does a herd of raindeer form a round shape when they sense on attack.

She started with this thought provoking idea that nature seeks some geometric optimization.

Prof. Anisa Chorwadwala *photo credits IISER Pune*

These stimulating mathematical thought she says have led to shape optimization problems, in general involve minimizing a cost functional. This variational problem involves solving a PDE she noted.

She discussed some historical problems also led to the development of subject.

Prof. Rukmini Dey (*Professor ICTS Bangalore*)

Prof. Rukmini Dey *photo credits ICTS Bangalore*

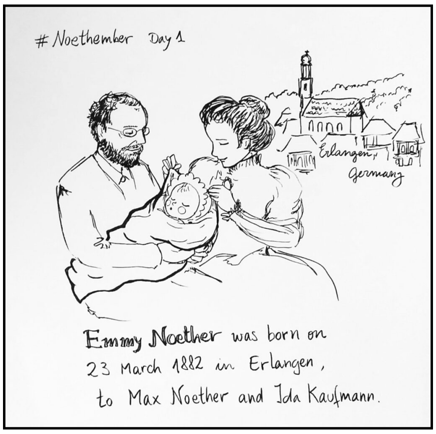
Professor Rukmini Dey of the International center for Theoretical Sciences Bangalore spoke on about the minimal surfaces and related complex analysis.

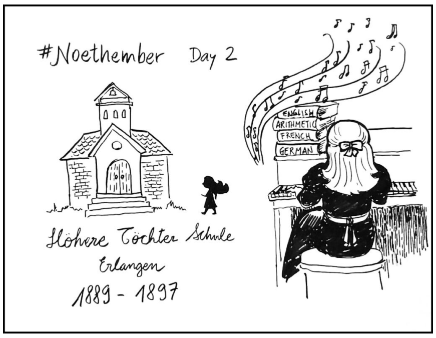
Minimal surfaces are surfaces that have the property that they locally minimize their area. This means that if you take a small piece of the surface, it will have the smallest possible area for that shape. Simultaneously she also defined maximal surfaces Maximal surfaces, are surfaces that have the property that they cannot be locally extended to any larger surface. From a differential geometric point of view Minimal surfaces in 3-d Euclidean space and maximal surfaces in 3-d Lorentz Minkowski space are defined to be zero mean curvature surfaces. Again from the complex analysis standpoint the general solutions for minimal and maximal solutions are obtained by the Weierstrass-Enneper representations of these surfaces. In her talk she showed how to first rederive the Weierstrass-Enneper representation of a minimal and maximal surface using hodographic coordinates which was introduced in the context of solitons by Barbishov and Chernikov. Interesting ideas from other areas of mathematics were also discussed by Prof. Dey. An interesting link between minimal surfaces and maximal surfaces and Born-Infeld solitons. Lastly she also provided the link about some identities we obtain from certain Euler-Ramanujan identities and their relation with some of these surfaces.

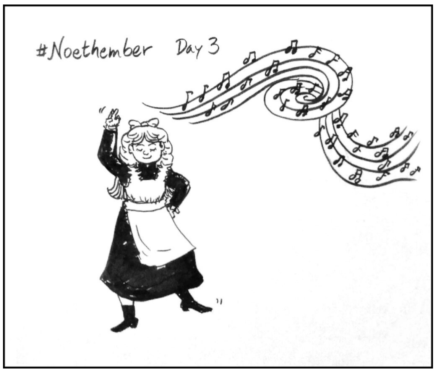
Under the banner of # Noethember, an international drawing challenge was organized in November 2018 to celebrate the life and work of mathematician Emmy Noether. The challenge was initiated by Constanza Rojas-Molina and aimed to encourage people to share their sketches illustrating facts about Noether's life and work on Twitter. The challenge lasted for 30 days, and people from all over the world participated by sharing their drawings on Twitter. The project resulted in a large number of drawings and a warm feeling among those who participated. The challenge was a great way to raise awareness about the contributions of Emmy Noether to the field of mathematics and to inspire people to learn more about her life and work.

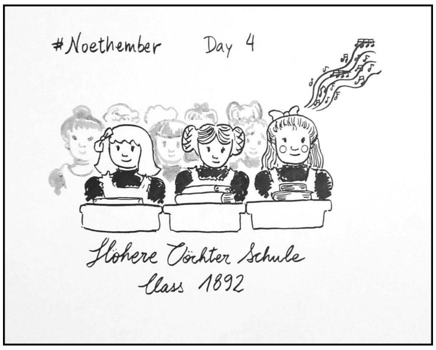
Given below are a few caricatures that have come from the 2018 (drawing competition):

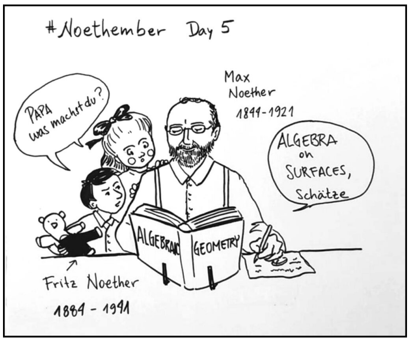
picture credits for all of these bhavana.org

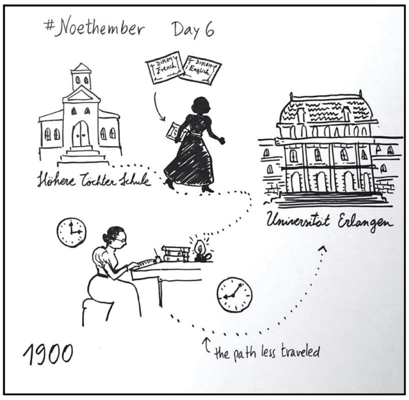


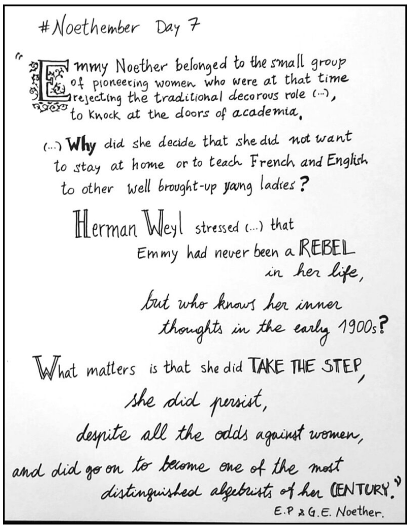


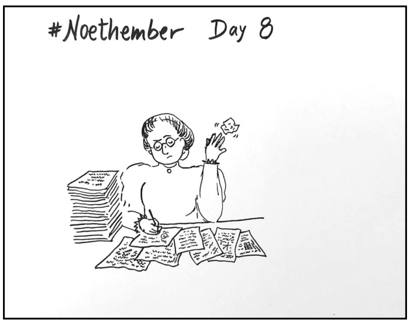


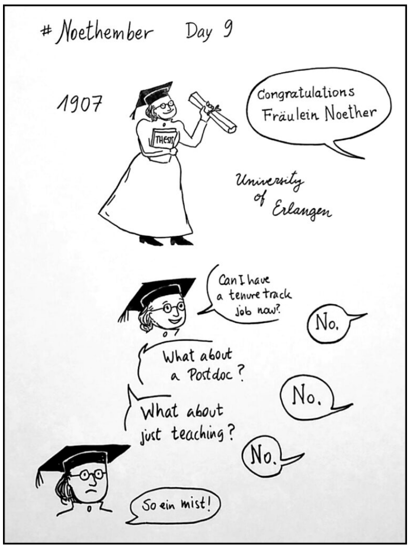


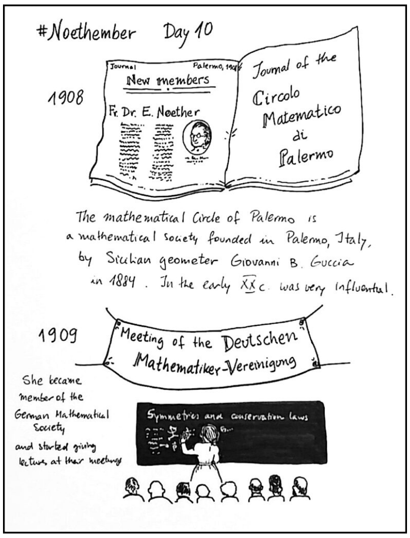

#Noethember Day 5
Max Noether 1844-1921
PAPA was machst du?
ALGEBRA on SURFACES, Schätze
ALGEBRA GEOMETRY
Fritz Noether 1884 - 1941


#Noethember Day 6
Höhere Tochter Schule
Universität Erlangen
1900
the path less traveled


#Noethember Day 7
"Emmy Noether belonged to the small group of pioneering women who were at that time rejecting the traditional decorous role (...), to knock at the doors of academia.
(...) Why did she decide that she did not want to stay at home or to teach French and English to other well brought-up young ladies?
Herman Weyl stressed (...) that Emmy had never been a REBEL in her life,
but who knows her inner thoughts in the early 1900s?
What matters is that she did TAKE THE STEP, she did persist, despite all the odds against women, and did go on to become one of the most distinguished algebrists of her CENTURY."
E.P & G.E. Noether.


#Noethember Day 8


#Noethember Day 9
1907
Congratulations Fräulein Noether
THESIS
University of Erlangen
Can I have a tenure track job now?
No.
What about a Postdoc?
No.
What about just teaching?
No.
So ein mist!


#Noethember Day 10
1908
Journal Palermo, 1908
New members
Fr. Dr. E. Noether
Journal of the Circolo Matematico di Palermo
The mathematical Circle of Palermo is a mathematical society founded in Palermo, Italy, by Sicilian geometer Giovanni B. Guccia in 1884. In the early XX c. was very influential.
1909
"Meeting of the Deutschen Mathematiker-Vereinigung"
She became member of the German Mathematical Society and started giving lectures at their meetings
Symmetries and conservation laws


#Noethember Day 11
Dear Fr Dr. Noether, your work is amazing! we need your help, please come to Göttingen. Big fans! Felix & David
Göttingen
Emmy, here!
Erlangen
Was? A Fräulein in our dear Faculty? Nein!
Faculty meeting

Variations on Conservation Laws for the Wave Equation

Cathaleen Synge Morawetz was a Canadian Mathematician known for her works in Partial differential equations of mixed type, aerodynamics, supersonic flows, and shock waves. She was the first woman to win the National medal of Science of the United states. Working under the supervision of Professor Kurt Otto Friedrichs, she started her career at MIT as a research associate and later returned to the CIMS at NYU. She was also the first woman to deliver the Gibbs Lecture of The American Mathematical Society.She died on August 8, 2017, in New York City. At the 1998 ICM held in Berlin, Morawetz delivered the Emmy Noether lecture entitled "Variations on Conservation Laws for the Wave Equation".
We first explain her work in some detail and also excerpted is a conversation with Professor Cathaleen Morawetz.

Cathleen Morawetz Photo courtesy of the Morawetz family

Cathleen Synge Morawetz was born on May 5, 1923, in Toronto, Canada. After receiving a bachelor's degree from the University of Toronto and a master's degree from the Massachusetts Institute of Technology, she received her doctorate from New York University in 1951. She was a Guggenheim Fellow during 1966-67 and 1978-79. In 1993 she was named *Outstanding Woman Scientist* by the Association of Women in Science.Apart from the Noether Lecture she also delivered the AMS Gibbs Lecture (1981) and an Invited Address of the Society for Industrial and Applied Mathematics (1982).
She is a member of the National Academy of Sciences. During 1995-96, Morawetz served as president of the AMS.
The citation for her National Medal of Science awarded in 1998 says "for pioneering advances in partial differential equations and wave propagation resulting in applications to aerodynamics, acoustics and optics". Further a press release archive of NSF says "In a series of three significant papers in the 1950's, Morawetz used ingenious new estimates for the solution of mixed nonlinear partial differential equations that ultimately led to advanced studies of wing design in aviation. In the early 1960s, she obtained important results in geometrical optics in connection with sonar and radar. It was known then that geometrical optics could be used to determine approximately the acoustic and electromagnetic fields scattered by objects. It was believed that this approximation became more accurate as the wavelength approached zero. Morawetz showed that this is the case and obtained an estimate of the error. Her result placed geometrical optics on a firmer foundationand led to further practical use of this approach."

Based on the above mentioned records and excerpts of interviews taken by several people the following **FACE-to-FACE** section is developed.

Qn: When asked about her childhood
CM:When I was very small and people were asking me what I wanted to be I would say I wanted to be a Mathematician, but if they asked me what is a "mathematician", I just had no clue then. Well, the most important thing probably in the point of view of my professor was that my father was a mathematician, John L. Synge. He did mathematics, physics, with

relativity, and he was Irish. My parents were both Anglo-Irish, they came from the south of Ireland but they were both Protestants. Except they both were not religious. And my father immigrated to Canada after his bachelor's degree in order to have a job and I was born in Canada. So was my older sister. We went back to Ireland, so I first went to school in Ireland when I was three years old. And we were there for five years, and then we came back to Canada. And then in 1948, no 1943, my parents moved to Columbus, Ohio but I left to work for the war effort in Quebec City. So I worked for a year in Quebec for an inspection board.

Qn: How old were you then?

CM:Well, I turned twenty-one while I was there. And then I came at the end of a year and I graduated from the University of Toronto in 1945.

Qn: What did you study there?

CM: I studied math there. In Toronto, the set up was that when you finished high school, you began to be eligible for scholarships and things. And the scholarship that I won required me to go into mathematics or chemistry. So I did that, and after three years, I got a little tired, and that's why I went to Quebec. But when I come back, I knew I had to finish up so I did that. So I majored in math but probably if I had been in America, I might have dropped out of math. You could change subjects which you could not do in Canada. So then, when I was graduating from Toronto, I ran into an old friend of my parents, who was professor in the math department, a woman. Cecilia Krieger, she never taught me, but she was a family friend.And when she found out that I had no plans, I was sitting here thinking going as a teacher to India she got all excited and said I had to go to graduate school. I looked into going into Caltech, but they didn't take any women. So then I applied to MIT. And I was interested in moving over and doing electrical engineering and when I went to MIT, I got a Canadian fellowship, and that was enough for tuition at MIT. And there I took some engineering courses, electrical engineering, and all I found that you have to know - you have to do the the arithmetic correctly! And I preferred more theoretical things. So I went back and finished up a master's degree with Professor Weisner and it was in elasticity. But in the meanwhile, I had become engaged and I got married in the middle of the term. So I wanted to leave MIT and go where my husband was, which was what I did. So I was only at MIT from the June of 1945 to January of 1946.

Qn: But while you were there, how did you feel about MIT? Was it very different?

CM: Well, I was just thinking about that and there was one very nice thing about MIT. First of all, they had always accepted women, as far as I know historically. But there were never very many of them. And sometime, some people had given money in the memory of their daughter to create a suite of rooms for the use of the women. There were no womens' dorms and certainly no undergraduate dorms at all. So we had a place where we could meet and it consisted of a living room, a bedroom with four beds, where you could take a snooze, a kitchen where you could eat or prepare food as you'd liked. So I met a lot of the girls there and the predominant field of study was architecture. There were a lot of girls in [architecture], and my sister studied architecture so we had a common bond. And that was very nice. That was a very positive thing that MIT had, this place that was just for women. But other than that, I was kind of lonely at MIT. I didn't make any friends in classes. When I came here, that was a big difference. The math group was very integrated with the faculty and that was a big change. But my experiences at MIT were, that I was a bit lonely and I think that the girls were too. As time went on, there were many girls, and I guess it's about more than 50 percent girls by now.

Qn: Okay, so after MIT, we hear that you went with your husband to the next chapter in your life.

CM: We came here - He was working for the Bakelite, which was part of Union Carbide in Toronto and he was transferred to New Jersey. And so I tried to get a job at Bell Labs, which was nearby. And I remember that very well ... They told me that, in so many words, that being a woman, I would go into the pool of college graduates, and the fact that I had a master's degree from MIT meant nothing. So I didn't want to do that at all, I was very annoyed and I consulted my father then and he suggested that I should look up the group at NYU, which was run by Richard Courant. I obviously don't remember exactly the timing of things because it seems to me that when I saw Courant, I came down from MIT, and he talked to me. Maybe I had that behind. But anyway, I was turned off from Bell labs when that happened. So then I found that it was a terribly long commute, I think it took me an hour and fifteen minutes on the Jersey Central. I only came in three days a week. But then I had my children and Courant was very accommodating about that. He let me work as much as I wanted to work. And actually, I finished my degree in 1951, when I did the last work with him. And last year, before I got my degree, 49-50, I had my second child. We lived in Brooklyn in a one-bedroom apartment and it was a little-it was difficult. I got pretty depressed, I had two kids, and not very good health, and I wanted to get my Ph.D. In the end, I did it, but I don't recommend that combination.

Qn: What was the world like back then? You said that you got your degree at NYU?

CM: Well, nobody did what I was doing. There were people who thought it was wonderful, there were people who would walk into my office and tell me I was destroying my children's lives. You know, people are very funny. But that just made me angry, depressed. I had four children and by the time the last one was in school, I'd been back to MIT for a postdoc and then I was appointed to the faculty here and secure in my life's ambitions. When Courant interviewed me, he invited to have lunch with everybody and there they used to discuss some great theorems, they discussed Segal's lectures on the packing of spheres and then they talked about some other theorems...I felt ah this is a great place.

Qn: How was it in New York then?

CM: Once I got in to the atmosphere of Courant (CIMS of the New York State University) there was a very positive atitude. Richard Courant himself had been a supporter of Emmy Noether years before and so the idea of a women having a profession was certainly accepted.

Cathleen Morawetz *Photo courtesy New York University*

Qn: We think it's very different hearing about this and comparing it to the way it is now.

CM: Well, nowadays, a lot of women work after they're married. No one considers giving up their work until children. But even then, it's still a question of the right way to go about it. I would not have liked to stay home. That certainly- I just certainly would not have liked that at all. So I didn't! And of course once I had settled in a kind of routine- well I didn't finish the story properly. So in 49, I had a second child. And then in 50, my husband got a postdoc and Harvard medical and I went up to Boston and I got a job with C.C. Lin. He was at MIT for many years. Chia-Chiao Lin. Eventually, he went to Florida and also to China-back to China. He's still alive, he's a very well-known man. He had been a student of my father's at one time, so I don't know if that was why he gave me the job but it was a good job. He was very particular. He would give me a problem to work on for two weeks, and if I didn't get anywhere with it in two weeks he would take it away and give me another problem. It went on like that for several months and the funny thing, then in connection with one of these problems, I brought in my own problem and he said, "That's the problem I gave you two months ago!" I worked in ordinary differential equations and I worked on two papers, I didn't publish my thesis, which I felt was incomplete. And those were my first two publications, the ones done under the supervision of C. C. Lin. And of course, when I went back to Boston, when I was living there for the second time, it was so very different. I was married, I had these two kids, I had to worry about getting help for taking care of these kids. I was very lucky that somebody arranged for me to get my little girl, who was then two and a half, into a very good nursery school and also got us an apartment in Brookline, so that the school was across the street. That really worked out very well for us. Then I got a very nice woman to - who looked after the little ones and the other one, who had been some kind of an off-duty nurse maid for the Kennedy's. She has rather precise ideas of what they should be doing at certain stages. But she was very good, very nice and warm. And all that settled down. I mean, I got good help, we had a rather nice apartment, a big floor there on Brookline, what is that big street there, I forget, passing through Brookline, near the Reservoir. At any rate, that was fine. And then after a year, during that year, the Polymer Institute of Brooklyn invited Herbert to be on the faculty, and Courant had told me-First, Courant told me that when I got my degree, that was it. And I said, "Of course, of course", I didn't expect to be kept, and then I went up to Boston and there was a meeting there and Courant came. He visited us and he said if I should come back to New York, he would have a job for me. That was really very nice, I wouldn't have to look for another job. So, then we came back to New York and we lived in New Rochelle, which is where a large number of people who worked here lived at that time. And I commuted three days a week, something like that, and I had a part-time salary, which was our agreement. In 1960, which was quite a bit later, I went on the faculty. But I never pushed to have the positions. I just, I felt that they made me a very good deal, I was able to work as much as I wanted to. And when I came back here, Wilhelm Behrens came on the faculty and he was doing a lot of work in fluid dynamics, air foil, so I got into that. And then I proved an important theorem for them, so that established that.

Qn: What were your major areas of work?

Well, the theorem is that if you take an air flow and you speed it up, it would be going supersonically if the whole thing was moving faster than the speed of sound. But before it gets to there, the flow at first is very smooth, there are no shock waves in it. Shock waves are bad because they absorb energy and they cause drag. At a certain time for a particular air flow, a little bubble forms and mostly that bubble has a shock in it. But there are certain air flows for which there is no shock. So the question was connected when do you get one and when do you get the other, what is the nature of the shock you get, and things like that. That was the first paper that I did something better than the others, the others were a little humdrum. This was an important paper. And then later I worked on plasma physics, that's because that was a project here. So I couldn't work on the project and not have any teaching to do. And that was very good for me. So I did plasma physics.

The other area that I worked on was scattering theory and that was inspired by a lecture by Joe Kelers. When I was listening to the fact that in one dimension it was like a hyperbolic equation and in another elliptic, It intrigued me and I went back after the lecture and started working and got some crucial identities within a few days, that's the only time I did something really fast. This work is about energy disturbed through electromagnetism and the decay seems to be exponential.

Qn:What is your favourite piece of work?

CM: Getting estimates for the wave equation. That was sort of surprisingly easy. Another is the paper on exponential decay of solutions of a wave equation where I broke it down to every instance of time and then it becomes easy for the proof of exponential decay.

Mathematical Results and Challenges in Learning Theory

Professor Ingrid Daubechies is a Belgian Mathematician and Physicist best known for her work on wavelets and image compression. She originally completed her PhD in Physics under the guidance of Professor Alex Grossmann at Vrije Universiteit Brussel. At Courant Institute of Mathematical Sciences (NYU) she made her best-known discovery based on quadrature mirror filter technology leading to the construction of compactly supported continuous wavelets. She worked in the Mathematics Department at Rutgers University, New Brunswick till 1994. Working at various places including Princeton University, she is currently holding mathematics professorship at Duke University. At the beginning of her career she joined Bell Labs in Murray Hills New Jersey. Her contributions to the field of mathematics and her work on wavelet theory have had a significant impact on digital imaging and communications. The name Daubechies is linked to a family of orthonormal wavelets called the biorthogonal CDF wavelet used in the JPEG standard. The 2006 Noether lecture organised by the association for Women in Mathematics was delivered by Professor Daubechies. The lecture was entitled "Mathematical Results and Challenges in Learning Theory".

The AdaBoost algorithm (Freund and Schapire, 1997) was designed to combine many "weak" hypotheses that perform slightly better than random guessing into a "strong" hypothesis that has very low error. Although extensively studied, some of AdaBoost's basic convergence properties are not fully understood. This open problem focuses on one of these, namely, the convergence of the distributions over training examples that are iteratively computed by the algorithm.

AdaBoost is shown in Fig. 1; see Schapire and Freund (2012) for further background. Briefly, we are given training examples $(\mathrm{x}_1, \mathrm{y}_1), \ldots, (\mathrm{x}_m, \mathrm{y}_m)$. On each of a sequence of rounds t, Ada- Boost computes a distribution D_t over the training set which is used to select a weak hypothesis h_t from some space $\mathcal{H} = \{\bar{h}_1, \ldots, \bar{h}_N\}$, which we presume to be finite and closed under negation (so that $-\mathrm{h} \in \mathcal{H}$ if $\mathrm{h} \in \mathcal{H}$). To simplify the discussion, we assume that each weak hypothesis is selected exhaustively, meaning that h_t is chosen, among all $\mathrm{h} \in \mathcal{H}$, to have minimum weighted error $\Pr_{\sim \mathbf{D}_t} [\mathrm{h}_t(\mathrm{x}_i) \neq \mathrm{y}_i]$, which is exactly equivalent to choosing h_t to have maximum weighted "correlation" r_t, as defined in the figure. The chosen weak hypotheses can eventually be combined into a final classifier H, as in the figure, although our focus here is only on the distributions D_t.

Each distribution D_t can be viewed as a point in $\mathbb{R}^m$, or more specifically, on the probability simplex. AdaBoost, together with an exhaustive choice of weak hypotheses, can be regarded as defining a deterministic mapping from one distribution D_t to the next distribution D_{t+1}.

Several authors (Rudin et al., 2004; Kutin, 2002; Amit and Blanchard, 2001) have independently noticed these distributions converging to cycles as t gets large. Such behavior is readily observed when the number of training examples and weak hypotheses is small. On the other hand, in the more realistic case of many examples and very many weak hypotheses, other authors have reported that AdaBoost's behavior can appear chaotic with respect to its distributions (Caprile et al., 2002).

In our experiments (Rudin et al., 2004), although the initial behavior may seem chaotic, the distributions tend to converge to a cycle. It is not known how to characterize the relationship of the examples and hypotheses to properties of the cycle, such as its length, which can vary substantially.Boosting is often studied under a weak learning assumption, which, in our set-up, states that the correlations r t are bounded away from zero (so that, for some $\mathrm{c} > 0$, we have $\mathrm{r}_t \geq \mathrm{c}$ on every round t). When this assumption does not hold, it was

Professor Ingrid Daubechies   Photo credits TWAS website

Given: (x_1, y_1),...,(x_m, y_m) where $x_i \in \mathcal{X}$,$y_i \in$ {-1, +1}
set $\mathcal{H} = \bar{h}_1,\ldots, \bar{h}_N$ of weak hypotheses $\bar{h}_j : \mathcal{X} \to$ {-1, +1}.
Initialize: $D_1(i) = 1/m$ for i = 1,...,m.
For t = 1,...,T

- Train weak learner using distribution $\mathbf{D}_t$; that is, find weak hypothesis $h_t \in \mathcal{H}$ with maximum correlation $r_t = E_{i\sim\mathbf{D}t}[y_i\ h_t(x_i)]$.
- Choose $\alpha_t = \frac{1}{2}$ ln $((1 + r_t)/(1 - r_t))$.
- Update, for i = 1,..., m: $D_{t+1}(i) = D_t(i)$ exp($-\alpha_t\ y_i\ h_t\ (x_i\ ))/Z_t$
  where Z_t is a normalization factor (chosen so that $\mathbf{D}_{t+1}$ will be a distribution).
  output the final hypothesis: $H(x)$ = sign $(\Sigma_{t=1}^{t}\alpha_t h_t(x))$

Figure 1: The boosting algorithm AdaBoost

shown by Collins et al. (2002) that the distributions $\mathbf{D}_t$ must converge to a single, uniquely-defined point (a degenerate cycle of length one). When the weak learning assumption does hold, the distributions cannot converge to a single point, but they still may converge to a cycle. Thus, the open problem is concerned with this latter case only.

If it were possible to show that AdaBoost's distributions always converge to a cycle, and if one could actually find the cycle (either analytically or numerically), we might be able to substantially speed up the algorithm by "jumping" to its asymptotic behavior. Or we might be able to directly solve for AdaBoost's asymptotic "minimum margin," perhaps yielding direct insight into its ability to generalize to unseen training examples.

The mapping of $\mathbf{D}_t$ to $\mathbf{D}_{t+1}$ induced by AdaBoost can be greatly simplified. To do so, we define the m x N matrix M by $M_{ij} = y_i\bar{h}_j\ (x_i)$, thus encoding which weak hypotheses $\bar{h}_j \in \mathcal{H}$ are correct on which training examples (x_i, y_i). More abstractly, M can be viewed as an arbitrary {-1, +1}- valued matrix. Given $\mathbf{D}_t$, AdaBoost's computation of $\mathbf{D}_{t+1}$ can then be written equivalently as:

1. $j_t = \text{argmax}_j(\mathbf{D}_t{}^T\ M)_j$.
2. $r_t = (\mathbf{D}_t{}^T\ M)_{jt}$.
3. $D_{t+1}(i) = D_t(i)/(1 + r_t\ M_{ijt})$ for i = 1,..., m.

In step 1, a column j_t is selected with maximum correlation, corresponding to the choice of weak hypothesis $h_t = h_{jt}$.

In step 2, the correlation r_t is computed.

And in step 3, the new distribution $\mathbf{D}_{t+1}$ is computed, here written in an explicit form that does not require further normalization.

We say that the distributions $\mathbf{D}_t$ converge to a cycle if there exist "cycle points" (distributions) $\hat{D}_1$,...,$\hat{D}_l$ such that $\mathbf{D}_{kl+b} \to \hat{D}_b$ as integer $k \to \infty$, for b = 1,...,l. Thus, the open problem is to determine if, for every matrix **M**, the distributions $\mathbf{D}_t$ necessarily converge to a cycle.

Note that the maximizing column in step 1 may not be unique, in which case it is necessary to assume that ties are broken in some consistent fashion; for concreteness, let us suppose they are broken by selecting the column whose index j is smallest. Further, this step breaks the probability simplex of distributions into regions based on which hypotheses would be selected for which distributions; the ties occur at the boundary of such regions. As a result of this step, the iterative map is highly discontinuous. This lack of continuity is the cause of much of the difficulty in working with this problem since it means that many of the classical results on dynamical systems are inapplicable. For instance, this is the primary reason why the well-known "Period Three Implies Chaos" result (Li and Yorke, 1975) does not apply. (In fact, there are matrices M where every distribution must converge to a 3-cycle.).

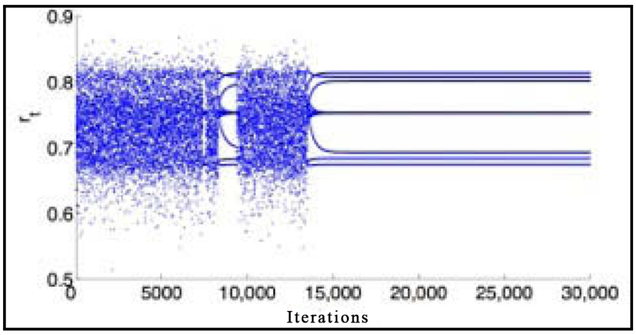


Fig 2:

A plot of r_t over 30,000 iterations of AdaBoost on a small matrix

From our previous studies (Rudin et al., 2004), we know there are some simple matrices M for which the distributions must converge to a cycle, and the iterated map provably forms a contraction in which nearby distributions $\mathbf{D}_t$ must get closer to the cycle points over time. (There are multiple possible cycles, but every possible distribution must converge to one of them.) Also, there is sometimes an analytical expression for these cycle points, and sometimes it is possible to prove there exists a unique solution for the cycle points if there is no closed-form solution.

In experiments on small matrices **M**, we have observed cycles of many different lengths, including odd and even lengths. Sometimes, AdaBoost takes a very long time to converge to a cycle. If one of the cycle points is close to the boundary between regions of the simplex, as the distribution is converging to the cycle, it could cross the boundary. At that point the distributions could map to a different part of the simplex altogether, and leave the region of attraction. This is illustrated in Fig. 2 where one of AdaBoost's parameters (r_t) is plotted over 30,000 iterations of AdaBoost. The apparent lines in the figure are made as AdaBoost alternates between a small number of possible values of r_t as it cycles. Around iteration 9,000, the weight vector crosses one of the regions in the simplex and no longer follows its previous cycle. Eventually, it finds this cycle and converges again. The open problem is to prove or disprove that AdaBoost's distributions $\mathbf{D}_t$ converge to a cycle in all cases, that is, for every {-1, +1}- valued matrix **M**. A reward of $100 is offered for a complete and general resolution of this problem.

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Moment Maps in stable Bundles: Where Analysis, Algebra and Topology Meet

Karen Uhlenbeck is an American mathematician currently serving as Professor Emeritus at the University of Texas, Austin. She also holds a distinguished professorship at the IAS, Princeton. She is one of the founders of modern geometric analysis and has made significant contributions to the fields of differential geometry, non-linear partial differential equations, and gauge theory. She completed her graduate studies and PhD at Brandeis University, where she was supervised by Prof. Richard Palais. In 1988 the Association of Women in Mathematics invited Professor Karen for the AWM-Noether lecture. The lecture was entitled "Moment Maps in Stable Bundles: Where Analysis Algebra and Topology Meet"

This is a time of finding interrelationships within mathematics. Or more bluntly, it is fashionable to study mathematical problems encompassing many areas of mathematics. This pleases me. As a student I had many crushes on various mathematics subjects, but sequentially. Now I can study them all at once. The gauge-theoretic study of stable holomorphic bundles on complex Kahler manifolds is a prototype of such crossroads. The subject of stable bundles is treated as an infinite-dimensional example of geometric invariant theory (as formulated by Mumford). Symplectic geometry plays an important role. The equations for the moduli space are the Yang-Mills equations (from physics). The group is the same gauge group from Yang-Mills, and the topology on the moduli space is studied via Morse Theory for the Yang-Mills functional.

Additional Comments: The gauge-theoretic study of stable holomorphic bundles is a field of mathematics that combines ideas from several different areas, including geometric invariant theory, symplectic geometry, and Yang-Mills theory.

A careful count includes not only analysis, algebra, and topology, but geometry and physics to boot! There is probably some number theory hidden somewhere in the second application below. Moreover, the list of possible applications is formidable. Best known are probably Donaldson's uses of these Yang-Mills moduli spaces to construct invariants of smooth compact four-dimensional manifolds. Nearly as important are the calculations of Atiyah-Bott on the topology of moduli spaces for stable bundles over curves. Corlette has used this machinery to classify flat bundles, and Simpson has employed it to study Hodge Strucun s. Atiyah and Hitchin have studied the interaction of magnetic monopoles (those things astronomers seem to think may exist). One can always guess this might be useful for string theory, which seems to be able to absorb every kind of mathematics. This is a lot of mathematics! Our graduate courses are not really designed to teach it, but graduate students seem to manage (better than me in fact) to absorb the necessary ideas.

Karen Uhlenbeck Photo credits ias website

Additional Comments: Thus the gauge-theoretic study of stable holomorphic bundles can be thought of as a sort of a junction point for some of these different areas of mathematics. It uses ideas from geometric invariant theory to study the moduli space of stable holomorphic bundles, and it uses ideas from symplectic geometry to define a natural metric on the moduli space. It also uses ideas from Yang-Mills theory to study the topology of the moduli space.

May I encourage more young women to try it. It is (comparatively) a woman's field. The list of references I include has a number of basic papers by women in it. I am sorry to leave the colorful slides out of the description of the talk, as well as the few mathematical concepts which were given. There is no good introductory survey. Perhaps I will find time to write a longer one sometime. None of the references is particularly expository.

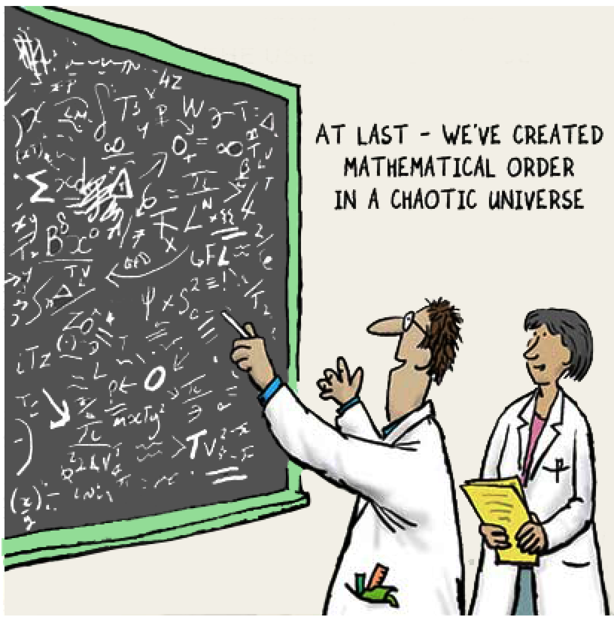
Uhlenbeck's most noted work focused on gauge theories. Her papers analysed the Yang-Mills equations in four dimensions, laying some of the analytical groundwork for many of the most exciting ideas in modern physics, from the Standard Model of particle physics to the search for a theory of quantum gravity. Her papers also inspired mathematicians Cliff Taubes and Simon Donaldson, paving the way for the work that won Donaldson the Fields Medal in 1986. Currently in New Jersey, Karen is constantly encouraging women mathematicians. A few of them are working closely to the topics cited in this lecture. For instance Prof. Nalini Joshi has been working on the moduli spaces of stable holomorphic bundles, and her work has helped to understand the topology of these spaces in the context of algebraic geometry.

Prof. Lisa Jeffrey is a professor of mathematics at the University of Toronto. She has worked on a variety of topics in gauge theory, including the Donaldson invariants and the Seiberg-Witten invariants.

Prof. Helen Verrill is a professor of mathematics at the University of California, San Diego. She has worked on the moduli spaces of stable holomorphic bundles, and her work has helped to understand the topology of these spaces.

Prof. Laura Fredrickson is a professor of mathematics at the University of California, Berkeley. She has worked on the Yang-Mills equations, and her work has helped to understand the relationship between these equations and the geometry of manifolds.

"Mathematics expresses values that reflect the cosmos, including orderliness, balance, harmony, logic, and abstract beauty." – Deepak Chopra



Symplectic Structures - A New Approach to Geometry
Dusa McDuff

Introduction

Symplectic geometry is the geometry of a closed skew-symmetric form. It turns out to be very different from the Riemannian geometry with which we are familiar. One important difference is that, although all its concepts are initially expressed in the smooth category (for example, in terms of differential forms), in some intrinsic way they do not involve derivatives. Thus symplectic geometry is essentially topological in nature. Indeed, one often talks about symplectic topology. Another important feature is that it is a 2-dimensional geometry that measures the area of complex curves instead of the length of real curves.

The classical geometry over the complex numbers is Kähler geometry, the geometry of a complex manifold with a compatible Riemannian metric. This is a very rich geometry with a detailed local structure. In contrast, symplectic geometry is flabby, though as should become clear, not completely flabby - there are interesting elements of global structure. The comparison can be roughly stated as follows:

$$\left\{\begin{array}{c} Kahler \\ rich\ \ detail \end{array}\right\} versus \left\{\begin{array}{c} symplectic \\ flabby,\ \ global \end{array}\right\}$$

In this article I will try to give an idea of symplectic geometry by comparing it with Kähler geometry. I will do this in three areas:

- Embeddings of round balls
- Structure of 4 -manifolds
- Properties of automorphisms

Basic Notions Let M^{2n} be a smooth closed manifold, that is, a compact smooth manifold without boundary. A symplectic structure ω on M is a closed ($d\omega = 0$), nondegenerate $\omega^n = (\omega \wedge ... \wedge \omega \neq 0)$ smooth 2-form. The nondegeneracy condition is equivalent to the fact that (w induces an isomorphism

$$\begin{array}{ccc} T_xM & \xrightarrow{\cong} & T_x^*M \\ X & \mapsto & t_X w \\ vectorfields & & 1-forms \end{array}$$

Basic Example. The form $\omega_0 = dx_1 \wedge dy_1 + ... + dx_n \wedge dy_n$ on Euclidean space $\mathbb{R}^{2n}$. In this case, the above isomorphism is given explicitly by the formulae

$$X = \frac{\partial}{\partial_{xj}} \mapsto l_X\omega_0 = dy_j$$

$$\frac{\partial}{\partial_{yj}} \mapsto -dxj.$$

Thus, if we identify both the tangent space $T_x\mathbb{R}^{2n}$ and the cotangent space $T_x^*\ \mathbb{R}^{2n}$ with $\mathbb{R}^{2n}$ in the usual way, viz:

$$\frac{\partial}{\partial_{xj}} \equiv e_{2j-1} \equiv dxj, \frac{\partial}{\partial_{yj}} \equiv e_{2j} \equiv dy_j,$$

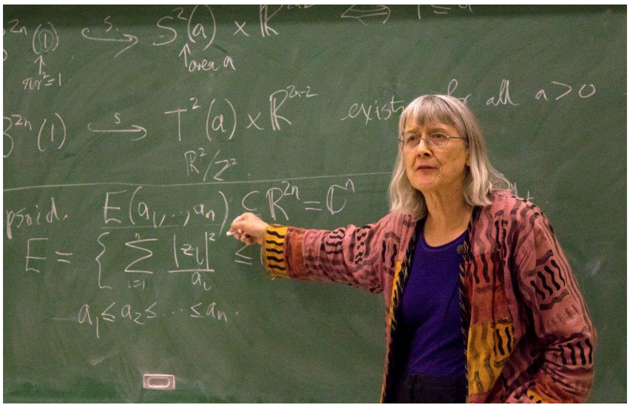
This isomorphism is a rotation through a quarter turn.

Every symplectic structure ω determines a volume form $\omega^n/n!$, that is, a non vanishing top-dimensional form that integrates to give a volume. In two dimensions, of course, ω is simply an area form. In higher dimensions it was suspected long ago that a symplectic structure is much richer than a volume form, but there was no hard evidence of this until the early 1980s, with Eliashberg's work on symplectic rigidity, the Conley-Zehnder proof of the Arnold conjecture for the torus, and Gromov's proof of the nonsqueezing theorem. We will discuss some of this below. For a much more detailed treatment of these questions and many further references the reader can consult [MS].

Here is the first main theorem on symplectic structures.

Theorem 1 [Darboux]. Every symplectic form is locally diffeomorphic to the above form ω_0.

Thus locally all symplectic forms are the same. In other words, all symplectic invariants are global in nature. It has turned out that, apart from obvious invariants such as the de Rham cohomology class $[\omega] \in H^2$ (M,**R**) of the symplectic form, it is hard to get one's hands on these global invariants, which is why symplectic geometry has taken so long to be developed. Another important fact that goes along with the local uniqueness of symplectic structures (one cannot exactly call it a consequence) is that a symplectic structure has a rich group of automorphisms. We discuss this further below.



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Symplectic structures have two main aspects: the geometric and the dynamic. We start with the geometric, the connection with Riemannian and Kähler geometry.

The Geometric Aspect

There is a contractible family of Riemannian metrics on M associated to ω which are constructed via ω-compatible almost complex structures J. Here J is an automorphism

$$\text{J: TM} \to \text{TM}, \qquad \text{J}^2 = \text{-Id}$$

that turns TM into a complex vector bundle. The compatibility conditions are:

$$\omega(\text{x,y}) = \omega(\text{Jx, Jy}),$$
$$\text{and } \omega(\text{x,Jx}) > 0 \ \forall \text{ x} \neq 0.$$

They imply that the bilinear form

$$\text{gJ : gJ (x,y)} = \omega(\text{x,Jy})$$

is a Riemannian metric. For each ω the set of such J is nonempty and contractible.

Examples

- The standard almost complex structure J_0 on $\mathbb{R}^{2n}$ given by
$$J_0\left(\frac{\partial}{\partial_{xj}}\right) = \frac{\partial}{\partial_{yj}}, \quad J_0\left(\frac{\partial}{\partial_{yj}}\right) = -\frac{\partial}{\partial_{xj}}$$
is compatible with ω_0.
- The almost complex structure J induced by the complex structure on a Kähler manifold.

There is an important difference between Kähler manifolds and symplectic manifolds. A Kähler manifold M has a fixed complex structure built into its points; M is made from pieces of complex Euclidean space C^n that are patched by holomorphic maps. One adds a metric g to this complex manifold and then defines the symplectic form ω_J by setting

$$\omega_J(x, y) = g(Jx, y).$$

(For this to work, g must be compatible with J in a rather strong sense: J has to be parallel with respect to the Levi-Civita connection in order for ω_J to be closed. Not all complex manifolds can be given a Kähler structure.)

On the other hand, a symplectic manifold first has the form ω, and then there is a family of J imposed at the tangent space level (not on the points). Note that the only intrinsic measurements that one can make on a symplectic manifold are 2-dimensional; i.e., if S is a little piece of 2-dimensional surface, then one can measure

$$\int_s \omega = area_\omega\ S.$$

It was the great insight of Gromov to realize that in symplectic geometry the correct replacement for geodesics are J-holomorphic curves. These are maps $u : (\sum, j) \to (M, J)$ of a Riemann surface $\sum$ into M that satisfy the generalized Cauchy-Riemann equation:

$$\mathbf{du} \circ j = \mathbf{J} \circ \mathbf{du}.$$

(Here j is the complex structure on the Riemann surface.) In fact, the image u($\sum$) is a minimal surface in M when it is given the metric gJ, so the analogy with geodesics is not far-fetched. There is a very nice theory of these curves-one application is mentioned below-and they occur as an essential ingredient in many symplectic constructions, for example, in Floer theory.

In his 1998 Gibbs lecture Witten discussed two "deformations" of classical physics, one to quantum theory and the other to string theory. I would like to propose that in some sense the passage from Riemannian (or Kähler) to symplectic geometry is analogous to these deformations. Symplectic geometry was of course first explored because of the fact that the classical equations of motion can be put in Hamiltonian form and that symplectic properties can be exploited to solve these equations incertain important cases. Therefore, because symplectic structures are built into the classical theory, they are very important in the new deformed theories. In "classical" symplectic geometry very little was understood about global topological properties of symplectomorphisms. Now, in both of Witten's deformations, new structures are being found that relate in some way to the new global symplectic geometry that is concurrently being developed.

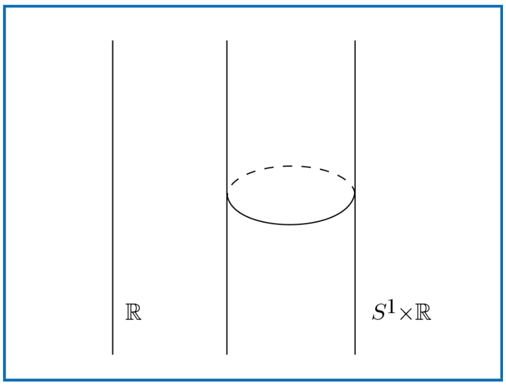

Fig1

I shall not say anything here about the problem of quantization (though recently Fedosov and Kontsevich have achieved great success with this question), but now briefly discuss the interconnection with string theory. Witten pointed out hat one basic consequence

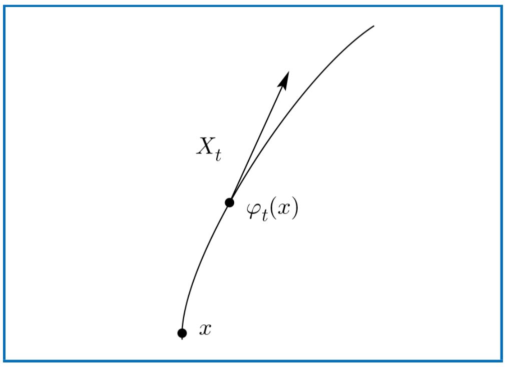

Fig2

of replacing the points that make up the configuration spaces of classical physics by strings (which in the closed case are just circles) is that the time line of a point-usually identified with the real line $\mathbb{R}$ - is replaced by the time line of a string, which is a cylinder S^1X $\mathbb{R}$; see Figure 1. This cylinder can be identified with the quotient $\mathbb{C}/\mathbb{Z}$ of the complex plane $\mathbb{C}$ by a translation and so has a natural complex structure. Thus the passage to string theory involves replacing $\mathbb{R}$ by $\mathbb{C}$ (or $\mathbb{C}/\mathbb{Z}$), and so going from a geometry in which 1-dimensional objects such as geodesics are of paramount importance to one in which 2-dimensional objects such as J-holomorphic curves are the crucial elements. It is no accident that some of the new ideas that have come into mathematics from physics (such as quantum cohomology and mirror symmetry) involve J-holomorphic curves in an essential way.

The Dynamic Aspect

As mentioned above, the nondegeneracy of the symplectic form ω is equivalent to the condition that there is a bijective correspondence

$$\begin{array}{ccc} T_xM & \xrightarrow{\cong} & T_x^*M \\ X & \mapsto & t_X\omega = \omega(X,.) \\ vector fields & & 1-forms \end{array}$$

The next important point is that the closedness of ω implies that the symplectic vector fields correspond precisely to the closed 1-forms. A vector field X is said to be symplectic if its flow ϕ Xt consists of symplectomorphisms, that is, if

$$(\phi_t^X t)^*\omega = \omega, \forall t.$$

Because

$$\frac{d}{dt}(\phi_t^X)^*\omega = (\phi_t^X t)^*(\mathcal{L}_X\omega),$$

X is symplectic if and only if $\mathcal{L}_x\omega = 0$ where $\mathcal{L}$x denotes the Lie derivative. The calculation

$$\mathcal{L}_x\omega = l_x d_\omega + d(l_x\omega) = d(l_x\omega)$$

shows that X is symplectic exactly corresponding 1-form α = Lx_w is closed. Since every manifold supports many closed 1-forms, the group Symp(M, ω) of all symplectomorphisms is infinite-dimensional. It has a normal subgroup Ham(M, ω) that corresponds to the exact 1-forms α = dH. By definition, $\phi \in$ Ham(M, ω) if it is the endpoint of a path $\phi_t \in$ [0,1], starting at the identity ϕ_0 = id that is tangent to a family of vector fields X$_t$ for which t((X$_t$)ω is exact for all t; see Figure 2. In this case there is a time-dependent function H$_t$: M $\to \mathbb{R}$ (called the generating Hamiltonian) such that t(X$_t$)ω = dHt for all t.
When the first Betti number b$_1$ = dimH^1 (M, $\mathbb{R}$)of M vanishes, Ham(M,ω) is simply the identity component Symp$_0$(M, ω) of the symplectomorphism group. In general, there is a short exact sequence

$$0 \to Ham(M,\omega) \to Symp_o(M,\omega)$$
$$\to H^1(M,\mathbb{R})/\Gamma_\omega \to 0$$

where the flux group Γ_ω is a subgroup of H^1(M,$\mathbb{R}$).

Example. In the case of the torus T^2 with a symplectic form dx $\wedge$ dy of total area 1, the group Γ_ω is H^1 (M, $\mathbb{Z}$). The family of rotations R$_t$: (x, y) $\to$ (x+t, y) of the torus T^2 consists of symplectomorphisms that are not Hamiltonian. Its image under the homomorphism to H^1(M, $\mathbb{R}$)/Γ_ω is the family of 1-forms t[dy].

It has recently been shown [LMP 1997] that Γ_ω has rank at most b$_1$. One interesting question here is whether the flux group Γ_ω is always discrete. This is equivalent to asking whether the group Ham(M, ω) is closed in the C^1-topology, that is, in the topology of uniform convergence of the first derivative. The group is discrete if

- the symplectic class [ω] $\in$ H^2(M,$\mathbb{R}$) is rational or
- If the map $\wedge$ [ω]$^{n-1}$: H^1 (M, $\mathbb{R}$) $\to$ H$^{2n?1}$ (M, $\mathbb{R}$) is an isomorphism.

Because of the hard Lefschetz theorem, this last case includes all Kähler manifolds.

The group Symp(M, ω) is a large and interesting group that contains a great deal of information. For example, Banyaga has shown that its structure as an abstract group uniquely determines the symplectic manifold (M, ω). In other words, if the groups Symp(M, ω) and Symp(N, σ) are isomorphic as discrete groups, then there is a diffeomorphism ϕ : M $\to$ N such that $\phi * \sigma = \omega$. We will describe some other results on the group of symplectomorphisms later. Meanwhile, here is a recent result that shows that $Symp(M,\omega)$ is significantly different from the group of all diffeomorphisms.

Proposition 2 [Seidel]. The natural map $\Pi_0(Symp(M,\omega))$ $\to$ Π_0(Diff(M)) is not injective in many cases.
For example, the natural map is not injective if M is a K3 surface. To prove this, Seidel constructs a symplectic Dehn twist t near a Lagrangian 2-sphere whose square is diffeotopic to the identity but not symplectically isotopic to the identity. There are other examples where the map $\Pi_k(Symp(M,\omega))$ $\to$ Π_k(Diff(M)) is not onto (for example, when M = S^2 x S^2).

Symplectic Embeddings of Balls

Gromov's Nonsqueezing Theorem

Consider a ball $B^{2n}(\mathbf{r})$ of radius r and a cylinder $\mathbb{Z}(\lambda) = B^2(\lambda) \times \mathbb{R}^{2n-2}$ of radius λ in standard Euclidean space $(\mathbb{R}^{2n}, \omega)$. Here it is important that the two coordinates (x_1, y_1) that span the disc $B^2(\lambda)$ are "symplectic", that is, $\omega_0(\frac{\partial}{\partial_{x1}}, \frac{\partial}{\partial_{y1}}) \neq 0$ The question is: when is there a symplectic embedding ϕ of the ball into the cylinder? Its answer is provided by Gromov's celebrated nonsqueezing theorem; see Figure 3.

Theorem 3 [Gromov]. There is a symplectic embedding of the ball of radius r into the cylinder of radius λ if and only if $r \leq \lambda$.

The idea of the proof is very roughly the following. For each ω_0-compatible almost complex structure J the cylinder has a slicing by J-holomorphic discs of area $\pi\lambda^2$. If the ball is embedded in the cylinder, this slicing will induce a slicing of the ball; but if J is suitably compatible with the embedding, this slicing of the ball has to have some slices of ω_0-area $\geq \pi$ r^2. Hence we must have $r \leq \lambda$.

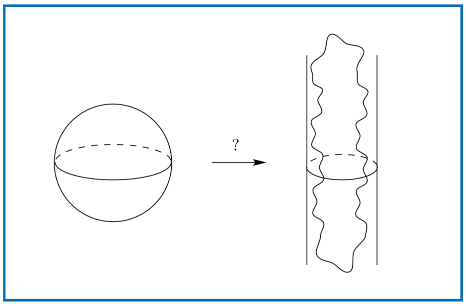

Fig3

This theorem underlies all of symplectic topology. As the following result shows, the nonsqueezing property characterizes symplectomorphisms. Darboux's theorem implies that if we want to find a criterion that characterizes general symplectomorphisms, it suffices to do this for symplectomorphisms of standard Euclidean space $(\mathbb{R}^{2n}, \omega_0)$. Define a symplectic ball (or cylinder) of radius r in $(\mathbb{R}^{2n}, \omega_0)$ to be the image of the standard ball (or cylinder) of radius r by a symplectic embedding. We will say that a local diffeomorphism ϕ has the nonsqueezing property if there is no symplectic ball B whose image $\phi(B)$ is contained in a symplectic cylinder with radius strictly less than that of B.

Theorem 4 [Eliashberg, Ekeland-Hofer]. If ϕ is a local diffeomorphism of $\mathbb{R}^{2n}$ such that both ϕ and its inverse ϕ^{-1} have the nonsqueezing property, then ϕ is either symplectic or antisymplectic, that is,

$$\phi * (\omega_0) = \pm\omega_0.$$

Since the nonsqueezing condition involves only the images ϕ(B) of balls B, it is easy to see that it is satisfied by any uniform limit of symplectomorphisms. Hence we find:

Corollary 5 [Symplectic rigidity]. The group Symp(M, ω) is closed in the group Diff(M) in the topology of uniform convergence on compact sets.

This is what I meant by saying in the first paragraph that symplectic geometry is intrinsically topological in nature. Not much is yet understood about symplectic geometry at this level.

Symplectic Packing

Suppose we want to embed k disjoint equal balls symplectically into a compact symplectic manifold (M, ω). What restrictions are there? One way to approach this problem is to define

$$v_k(M, m) = sup\frac{Vol\ (k\ disj.\ equal\ balls\ in\ M)}{Vol(M, \frac{\omega}{n!})}$$

We say that (M, ω) has a full packing if v_k (M, ω) = 1; otherwise, there are packing obstructions. See Figure 4.

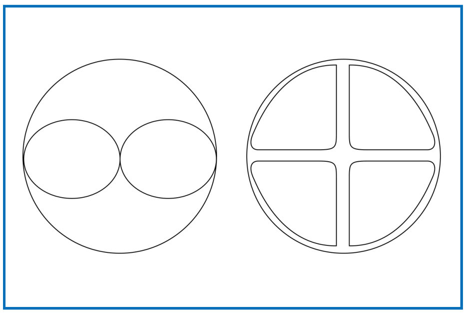

Fig4

One example that has been fully worked out is the case when M is the complex projective plane $\mathbb{C}P^2$ with the standard Fubini-Study metric. (Equivalently one could take M to be the unit ball B^{2n} (1)in $\mathbb{R}^{2n}$). In this case, results of Gromov, McDuff-Polterovich, and Biran show that $v_k(M, \omega)$ is as follows:

1	1	3	4	5	6	7	8	≥ 9
$v_k(M,\omega) = 1$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{20}{25}$	$\frac{24}{25}$	$\frac{63}{64}$	$\frac{288}{289}$	1

The result that v_k (CP^2) = 1 for all $k \geq 9$ is due to Biran [B].

Biran has also shown that for every symplectic 4-manifold there is an integer N such that

$$v_k(M, \omega) = 1 \; for \; k \geq N.$$

He proves this by showing that for all $\epsilon > 0$ there is a subset $V\epsilon$ of M such that M-$V\epsilon$ can be identified with a disc bundle over a Riemann surface with a standard symplectic form. Then he shows how to fill this disc bundle with balls. The existence of this disc bundle uses the deep work of Donaldson mentioned below, as well as an "inflation" technique of Lalonde-McDuff that allows one to change the symplectic form so that its volume is concentrated near the submanifold.

Thus symplectic packing is basically flabby: with enough balls one can maneuver them into shapes that fill the whole space. It is not known whether the analogous problem in the Kähler category is similarly flabby. Here one considers embeddings that are suitably compatible with both the holomorphic and the symplectic structure on M so that there is a corresponding Kähler form on the blow-up. It is not hard to show that the above calculations for $v_k(CP^2)$ apply also to Kähler embeddings if $k \leq 9$. Also, one can show that the Kähler equivalent $v_k^K(CP^2)$ of $v_k(CP^2)$ takes the value 1 whenever $k = d^2$. However, it is unknown if $v_k^K(CP^2) = 1$ for all $k > 9$. This question is related to difficult conjectures about Seshadri constants and about the structure of holomorphic curves on a generic blow-up of CP^2. Biran has recently obtained some interesting lower bounds for the numbers $v_k^K(CP^2)$ that involve continued fraction expansions. However, it is as yet unknown whether the appearance of these numbers is an artifact of his construction methods or whether they reflect something intrinsic to the problem.

Symplectic 4 -Manifolds

In this section we discuss some recent results on the existence of symplectic and Kähler structures on closed and connected 4-manifolds. This question is still not fully understood. The topological properties common to all manifolds with a particular geometric structure can be thought of as a large-scale global expression of this structure. Thus Donaldson's theorem that every symplectic 4-manifold has a blow-up that supports a generalized symplectic fibration is an illustration of how important fibered structures are in symplectic geometry. Fibered structures also arise when one is trying to construct the most economical embeddings of balls.

We begin with some general remarks that contrast symplectic with Kähler 4-manifolds.

- It has been known for a long time that there are non-Kähler symplectic manifolds. The first example was known to Kodaira and later rediscovered by Thurston. Here M is the nilmanifold obtained by quotienting out R^4 by the discrete group Γ that is generated by unit translations in the first three directions together with the map

$$(x, y, s, t) \mapsto (x, x + y, s, t + 1).$$

The symplectic form $dx \wedge dy + ds \wedge dt$ descends to a form ω on M. Note that M can also be considered as made from the manifold $T^2 \times S^1$ x [0, 1] by identifying the point (x, y, s, 0) with (x, x + y, s, 1). Therefore the projection (x, y, s, t) $\mapsto$ (s, t) induces a map from M onto the torus T^2 whose fiber is also a torus. The monodromy (or attaching map) of this fibration has the formula (x, y) $\mapsto$ (x + y, y) . This is an area-preserving and hence symplectic map but is not holomorphic. Therefore M has no obvious Kähler structure. In fact, it is easy to see that the first cohomology group H^1 (M, $\mathbb{R}$) has dimension-3. This implies that M has no Kähler structure at all because of the well-known fact that the odd Betti numbers $\dim H^{2k+1}$ (M, $\mathbb{R}$) of every Kähler manifold must be even. Indeed, $\dim H^{2k+1}$ can be written as p,q, which is even when a sum p+q=2k+1 dimH p + q is odd since $\dim H^{p,q} = \dim H^{q,p}$.

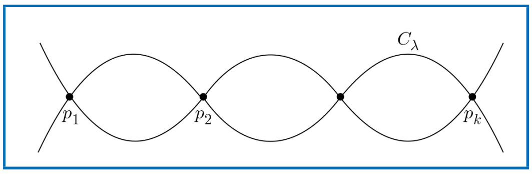

Fig5

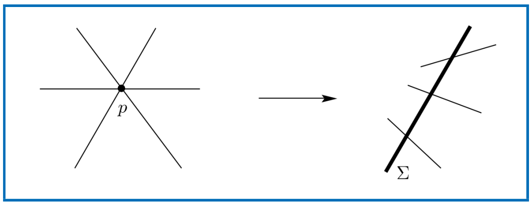

Fig6

- Gompf showed in 1994 that for any finitely presented group G there is a closed symplectic 4-manifold (M^4 , ω) with fundamental group G. On the other hand, there are restrictions on $\pi_1(M)$ if M is Kähler. For example, the remarks above imply that if M has dimension-4, we need the rank of $H_1(M) = G/[G,G]$ to be even. (There are other more subtle restrictions as well, which are at present not very well understood.)

- Taubes's structure theorem (1995-96) for the Seiberg-Witten invariants of symplectic 4-manifolds shows that some important features of the Kähler case persist in the symplectic case. Using this result, Szabo and then Fintushel-Stern constructed simply connected nonsymplectic 4-manifolds with nonzero Seiberg-Witten invariants. It follows that the class of symplectic 4-manifolds is strictly larger than the class of 4-manifolds with Kähler structure and strictly smaller than the class of 4-manifolds with nonzero Seiberg-Witten invariants. It is still not understood exactly what the class of symplectic 4-manifolds is. However, as the next result shows, symplectic 4-manifolds can be considered as a kind of flabby deformation of Kähler surfaces.

- It has been known for a long time that algebraic manifolds have blowups that support Lefschetz fibrations. Since the complex structure on every Kähler surface can be slightly deformed to be algebraic, it follows that every smooth 4-manifold that has a Kähler structure also supports a Lefschetz fibration.

Donaldson has recently (1997) shown that every symplectic 4-manifold has a blowup that has the structure of a symplectic Lefschetz fibration. Philosophically this is akin to showing that every 3-manifold has a Heegaard splitting: in other words, it is a general structure theorem that as yet does not make clear all topological properties of these manifolds. In view of the importance of this result we will spend some time explaining it.

Lefschetz Fibrations

Let M $\subset$ CP^N be an algebraic surface. Cut M by a pencil Pλ, $\lambda \in CP^1$, of hyperplanes with axis $A = CP^{N-2}$. (Here P_λ is just the set of all hyperplanes through A.) This gives a family of subvarieties $C_\lambda = M \cap P_\lambda$ that all go through the set M $\cap$ A; see Figure5. Since M has complex dimension-2 (and so real dimension-4), the set M $\cap$ A is a finite collection of points-presuming that A is generic-and the C_λ are complex curves that are nonsingular for all but a finite number of λ. Moreover, for generic A, the points in M $\cap$ A will be nonsingular on all the curves C_λ so that one can make the C_λ disjoint by blowing up these points; see Figure 6.

In this way one gets a family $\tilde{C}_\lambda$ curves on the blown-up manifold $\tilde{M}$, and the map

$$\begin{array}{cccc} \tilde{f}: & \tilde{M} & \to & CP^1 \\ & x \in \tilde{C}_\lambda & \mapsto & \lambda \end{array}$$

is a singular holomorphic fibration; see Figure 7.

Example. Let $C_i = \{\gamma_i\}$ for i = 0, 1 be two generic conics in CP^2. For $\lambda = [\lambda_0 : \lambda_1] \in CP^1$ define

$$C_\lambda = \{\lambda_0\gamma_0 + \lambda_1\gamma_1 = 0\}.$$

This gives a family of conics, all of them nondegenerate except for three pairs of lines.

Theorem 6 [Donaldson, 1997]. Every symplectic 4-manifold M has a blowup $\tilde{M}$ for which there is a smooth map = f : $\tilde{M} \to CP^1$ such that the following holds

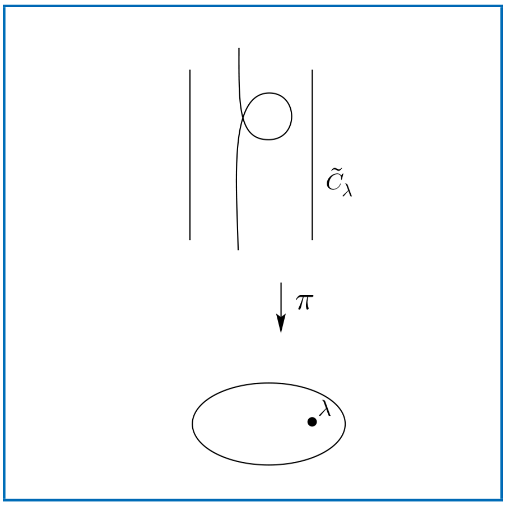

Fig7

- All but finitely many fibers of f are symplectically embedded submanifolds.
- The remaining fibers are symplectically immersed with just one double point. Moreover, a neighborhood of each of these singular fibers has a compatible complex structure.

Thus one can think of f as a complex Morse function, with singularities modelled on the most generic singularities in the holomorphic case. In particular, the monodromy around each singular fiber is given by a Dehn twist. In the complex case the singularities must satisfy subtle global compatibility conditions that are not fully understood. However, there are no such conditions in the symplectic case. If $f : M \to CP^1$ is a singular fibration as above such that the fibers support a smooth family of cohomologous symplectic forms that are compatible with the local structure near the singular fibers, then there is a compatible symplectic form Ω on M provided only that there is a cohomology class $a \in H^2(M)$ that restricts on the fibers to the class of the symplectic form.

To prove this theorem, Donaldson develops an "almost holomorphic" analysis that allows him to mimic the proof for algebraic manifolds. Very recently, he completed the generalization of this argument to higher dimensions, showing that every closed symplectic manifold has a suitable blowup that supports a symplectic Lefschetz fibration; also Auroux [Au].

Groups of Automorphisms

We come to the last of the areas in which I am contrasting symplectic with Kähler geometry. The group $Symp_0(M, \omega)$ of all symplectomorphisms of M that are symplectically isotopic to the identity was introduced earlier. I will write $Iso_0(M, J, \omega)$ (or simply $Iso_0(M)$) for the identity component of the group of isometries of the (closed) Kähler manifold (M, J, ω) when this is provided with the corresponding metric g_J. It is well known that this is a compact Lie group (often trivial). Further, because the symplectic form ω on a Kähler manifold is harmonic with respect to the Kähler metric and because a harmonic form is unique in its cohomology class by Hodge theory, the form ω is preserved by all isometries that fix its cohomology class $[\omega]$. Hence all elements of $Iso_0(M)$ preserve ω and therefore also preserve the complex structure J.

Some 4-Dimensional Examples

First of all, let me describe some cases in which these two groups are closely related. Note that they can never be equal, since $\mathrm{Symp}_0(M, \omega)$ is infinite-dimensional.

- If the complex projective plane CP^2 is given itsstandard structure, $\mathrm{Iso}_0(CP^2)$ is the projective unitary group PU(3), while $\mathrm{Symp}_0(CP^2, \omega)$ deformation retracts to PU(3).

- Let ω^λ be the symplectic form $(1 + \lambda)\sigma_0 \oplus \sigma_1$ on S^2 x S^2, where $\lambda \geq 0$ and where the σ_i are area forms on S 2 of area 1, and let J_{split} be the product almost complex structure. Then $\mathrm{Iso}_0(S^2 \times S^2, J_split, \omega^\lambda)$ is the product SO(3) $\times SO(3) forall \lambda$. On the other hand, Gromov (1985) proved that $\mathrm{Symp}_0(S^2 \times S^2, \omega^\lambda)$ deformation retracts to SO(3) x SO(3) if and only if $\lambda = 0$. Moreover, it has been shown by Abreu (1997) and Abreu-McDuff that $\mathrm{Symp}_0(S^2 \times S^2, \omega^\lambda)$ does not have the homotopy type of a compact Lie group when $\lambda > 0$. In fact, when $k - 1 < \lambda \leq k$, this group incorporates the isometry groups of the k + 1 different complex structures $J_0 = J_{split}, J_1, ..., J_k$ on $\mathrm{S}^2 \times S^2$ that are compatible with the Kähler form ω^λ. Similar results are true for the blowup of CP^2 at one point. However, nothing similar is known about most other manifolds, even one as simple as T^4.

It is obviously unreasonable to expect that the symplectomorphism group would be homotopy equivalent to the group of Kähler isometries in general. However, the next part of the discussion aims to show that some features of the Kähler case do persist in the general case.

The Group of Hamiltonian Symplectomorphisms

Let us write $\mathrm{HIso}(M)$ for the intersection of the isometry group Iso(M) with the group $\mathrm{Ham}(M, \omega)$ of Hamiltonian symplectomorphisms. The Lie algebra of $\mathrm{HIso}(M)$ may then be identified with a finite-dimensional space of smooth functions H on M, normalized by the condition that the mean $\int_M H\omega^n$ is zero. (As always, we assume that M is closed, that is, compact and without boundary.) Moreover, the exponential map is just the time one map of the corresponding flow:

$$exp : H \mapsto \phi^H = \phi_1^H.$$

Since the exponential map is surjective when the group is compact, it follows that every element ϕ of $\mathrm{HIso}(M)$ is the time one map ϕ^H of a Hamiltonian function $H : M \to \mathbb{R}$. Now, every critical point of H gives rise to a fixed point of ϕ^H, since the generating vector field X_H of the flow ϕ_t^H satisfies the equation $\mathrm{t}_{X H}\omega = dH$ and so vanishes at such critical points. It follows that for every $\phi \in \mathrm{HIso}(M)$ the number of its fixed points is at least as great as the number of critical points of a generating Hamiltonian H. Thus

$$\#Fix\phi \ \geq \ \#CritH \geq \textstyle\sum_i dimH^i(M, \mathbb{R}),$$

$$for\ all\ \phi \in \mathrm{HIso}(M).$$

Arnold's famous conjecture is that the above statement remains true for every Hamiltonian symplectomorphism whose fixed points are all non-degenerate. This was finally proved in 1996 for all symplectic manifolds by the combined efforts of many mathematicians, among them Floer, Hofer Salamon, Fukaya Ono, and Liu Tian. Thus:

Theorem 7 [Arnold's conjecture]. If (M, ω) is any compact symplectic manifold and $\phi \in \mathrm{Ham}(M)$ has no degenerate fixed points, then

$$\#Fix\phi \ \geq \textstyle\sum_i dimH^i(M, \mathbb{R}),$$

Note that it is essential here that ϕ be Hamiltonian. For example, the rotation (x, y) $\mapsto$ (x + t, y) of the torus T^2 is a non-Hamiltonian symplectomorphism with no fixed points.

Hamiltonian Loops

Our final result concerns a curious and recently discovered property of Hamiltonian loops. First observe that any loop $\{\phi_t\} \in \mathrm{Diff}(M)$ generates a homomorphism

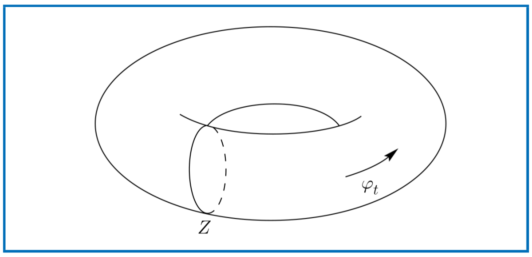

Fig8

$$\partial_\phi : H*(M) \to H_{*+1}(M)$$

that takes a k-cycle Z in M to the $(k+1)$-cycle $S^1 xZ \to M$ swept out by the action

$$S^1 xZ \to M : \quad (t, x) \mapsto \phi_t(x)$$

See Figure 8. Clearly, the map $\partial\ \phi$ depends only on the homology class of the loop $\{\ \phi_t\ \}$ in the space of continuous self-maps of M.

This map $\partial\ \phi$ can be expressed geometrically in terms of symplectic fibrations. Given a loop ϕ_t of symplectomorphisms of M , one can construct a fibration $\mathrm{P}_\phi \to \mathrm{S}^2$ with fiber M by thinking of ϕ_t as a clutching function, viz:

$$\begin{array}{ccc} P_\phi & = & M \times D^+ \cup_{\phi_t} M \times D \\ \downarrow & & \downarrow \\ S^2 & = & D^+ \cup D \ . \end{array}$$

It is not hard to show that the loop ϕ_t is isotopic to a Hamiltonian loop exactly when there is a symplectic form Ω on P_ϕ that restricts to the form ω on each fiber M. Further, the map $\partial_\phi : H_k(M) \to H_{k+1}(M)$ is precisely the boundary map in the Wang exact sequence for the fibration $P_\phi \to \mathrm{S}^2$.

Recent work of Lalonde-McDuff-Polterovich [LMP], which builds on ideas of Seidel, has shown that the map ∂_ϕ vanishes identically on rational homology when ϕ is a Hamiltonian loop. Thus we have the following result.

Proposition 8 [LMP]. If $(P_\phi,\ \Omega)$ is a symplectic manifold that fibers over S^2 with symplectic fiber (M, ω), then there is a vector space isomorphism

$$H^*(P_\phi, \mathbb{Q} \cong H^*(M, \mathbb{Q}) \otimes H^*(S^2, \mathbb{Q}).$$

This result generalizes in the Kähler case. Let us say that a fibration M $\to$ P $\to$ B with the property that H^* (P_ϕ, $\mathbb{Q}$) is additively isomorphic to $H^*(M, \mathbb{Q}) \otimes H^*(S^2, \mathbb{Q})$ is cohomologically split.Then Deligne showed that every holomorphic submersion from a Kähler manifold P to a base manifold B is cohomologically split. It is not yet known whether a similar result holds in the symplectic case, although there is a good notion of Hamiltonian fibration that generalizes the idea of a holomorphic submersion. (This is explained in the new edition of [MS] as well as in forthcoming work by [LMP].) The fact that at least some of these results on fibrations carry over to the symplectic case is yet another indication both of the naturality of fibered structures in symplectic geometry and of the special nature of Hamiltonian symplectomorphisms.

Professor Dusa McDuff receiving Doctor of Science, honoris causa - at University of St Andrews

This lecture note is based on AWM Noether Lecture delivered by Dusa McDuff which is available on ARXIV

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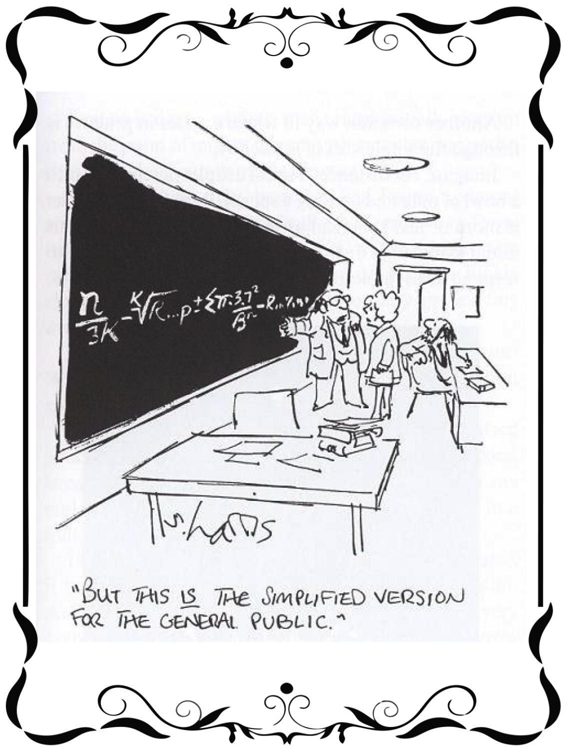

About the back cover.

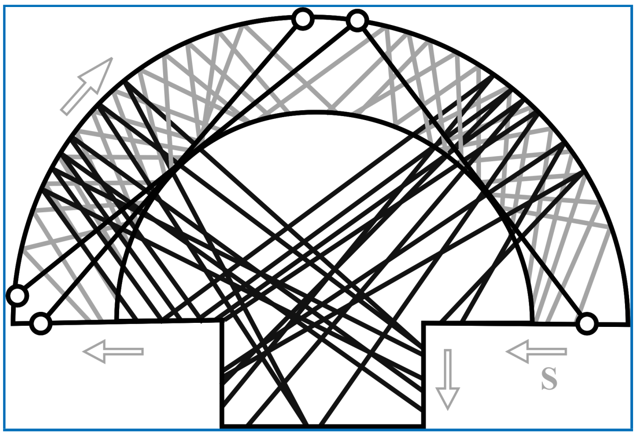

Photo Inspired by AMS Notices Article on Mushroom Billiards

The Back cover image is representative of a particular type of Billiard dynamical system called the mushroom billiard(with a semicircular cap).

The mathematical idealization of a (classical) billiard, is a type of Hamiltonian system consisting of confined point particles colliding elastically against the boundaries of a container of some shape . These mathematical objects have long held the interest of mathematicians, statistical mechanists and theoretical physicists for quite a long time leading to numerous advances in ergodic theory and dynamical systems. Billiards are studied in a continuous time setting and most often it is a deterministic system meaning they follow prescribed rules to evolve a certain state in time. So Billiards are Hamiltonian idealizations of the game of Billiards but the region contained by the boundary can have shapes other than rectangular and even be multidimensional. Mushroom billiards have a mixed phase space, meaning that they exhibit both regular and chaotic dynamics. The phase space is divided into regions with different types of motion, such as periodic orbits and chaotic trajectories. Mushroom billiards can exhibit sticky dynamics, which refers to the phenomenon where particles tend to stay in the chaotic region for longer periods of time due to the presence of marginally unstable periodic orbits. The relatively simple geometry of mushroom billiards makes them a useful tool for studying the interplay between regular and chaotic motion in dynamical systems. Their precise analysis can provide insights into the behavior of more complex systems. The simplest mushroom billiards consisting of a semicircular cap with a stem of some shape attached to the cap's base, provide a continuous transition between integrable (semi)circular billiards and ergodic (semi)stadium billiards as the stem width is increased from zero to the diameter of the circle. There are a large variety of admissible stem shapes but we can already make significant statements with regard to rectangular ones. Bunimovich [1] proved that for circular mushrooms , trajectories that remain in the cap are integrable, whereas those that enter the stem are chaotic (except for a set of measure zero). This gives a precise, complete characterization of mushroom billiard trajectories. One can take advantage of this knowledge in several ways. Changing the dimensions of a mushroom billiard controllably alters the relative volume fractions in phase space of initial conditions leading to integrable and chaotic trajectories. Using these mushrooms, Bunimovich has also shown that the stationary distribution of non-interacting particles in a container can be non- uniform.

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Down The Memory lane

Time Line Year - March 8, 2017

In this corner of MSIB Newsletter, described are some awesome nostalgic instances where there was a cause of celebration at MSIB.

MSI inaugurated the women in Mathematics programme on this day which was carefully chosen to be International Women's Day. Women mathematicians were invited for this purpose. Three women mathematician spoke on their field of expertise, one was Anisa Chorwadwala Assistant Professor at the Department of Mathematics, IISER, Pune. She spoke on maximum principles in analysis and geometry and shape optimization through differential geometry methods geometry methods. We would like the readers to note the connection of this kind of work with that of the Chern medel awardee (and Abel Laureate) Louis Nirenberg.

Dr. Arathi Sudarshan Head, department of Mathematics Jain-deemed-to-be University. Prof. Arathi spoke on mathematical modeling of blood pumping by the four chambers of the human heart. This project is accomplished by considering the concerned differential equations and related numerical schemes the speaker elaborated. The third speaker was Prof Rukmini Dey of the ICTS the International Center for Theoretical Sciences. She spoke on her particular field of interest namely minimal surfaces.

In the 'Math Fest' Programme to be held in December (National Mathematics Day - 22nd Dec 2020) we will be having a special session on Women in Mathematics.

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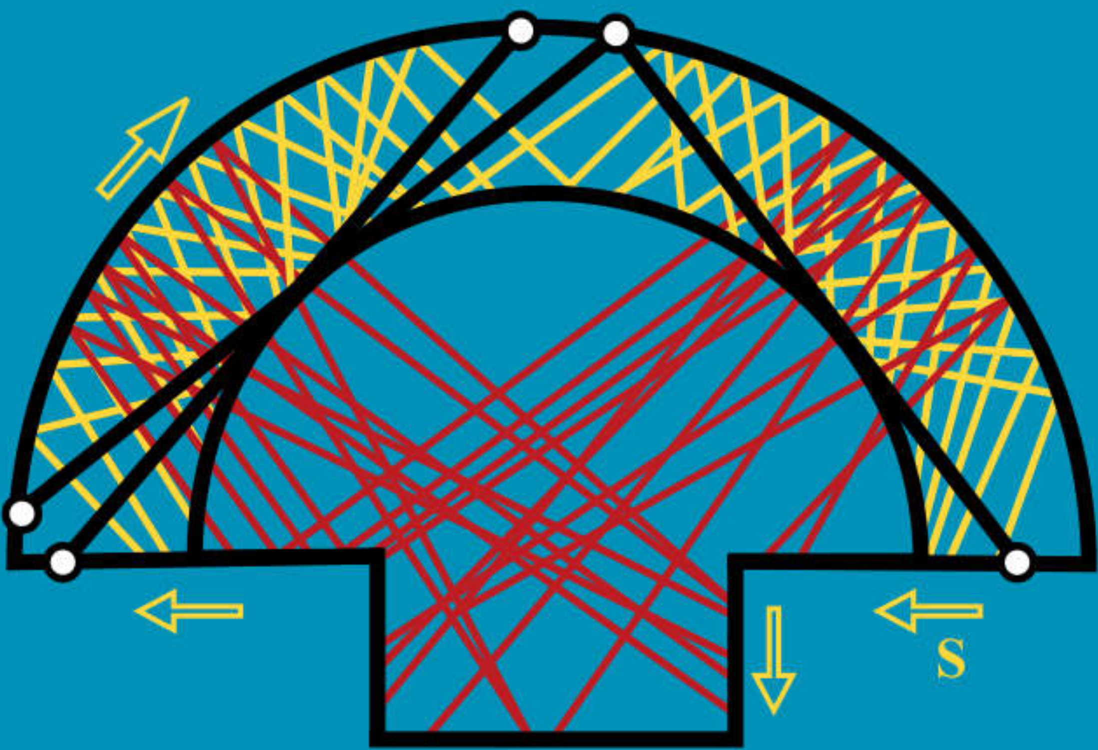

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